**LETTER** *Special Issue on the 1991 IEICE Fall Conference*

# Consideration on Adaptive System Identification Based on Filter Banks

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**SUMMARY** In some adaptive filtering applications, some researchers have explored the use of the multirate technique to improve the performance of the adaptive algorithms. However, some of these approaches have shown a degradation in performance. In this paper, we consider the conditions for filter banks which enable us to avoid the degradation. Next, we propose a filter bank with frequency sampling filters, and its performance is compared to those of conventional filter banks for adaptive system identification.

## 1. Introduction

In some adaptive filtering applications, conventional FIR filters can have many coefficients, resulting in a large computational complexity and slow the convergence of the algorithm. Thus, recently, some researchers have explored the use of the multirate technique in such applications, by dividing the input and desired signal into subbands(1)–(6). These approaches can improve the performance of the adaptive algorithm, while at the same reducing the computational complexity. However some of these have shown a degradation in performance, due to the introduction of either aliasing or spectral gaps in the output.

In this paper, first we discuss the reason why the degradation is shown in the performance, by means of obtaining the input-to-output relationship of the structure for adaptive system identification using filter banks. As a result, we show that the conditions for perfect QMF (Quadrature Mirror Filter) banks do not correspond to those for adaptive filtering in subbands.

Next, we describe that a filter bank based on frequency sampling filters is effective in avoiding the degradation. Our proposed bank is compared to some of conventional QMF banks for adaptive system identification.

## 2. Adaptive System Identification with Filter Bank

First, we describe that it is impossible to apply maximally decimated QMF banks to adaptive system identification in the strict sense, even if we use a perfect QMF bank.

In order to simplify the expression, we assume a two-band filter bank. Figure 1 shows a typical structure of a two-band filter bank. In general, the relationship between input $X(z)$ and output $Y(z)$ is given by

$$Y(z)=1/2\{H_0(z)F_0(z)+H_1(z)F_1(z)\}X(z)$$
$$+1/2\{H_0(-z)F_0(z)+H_1(-z)F_1(z)\}$$
$$\cdot X(-z). \qquad (1)$$

Suppose that Eq. (1) meets the following conditions, referred to as the necessary and sufficient condition for two-band perfact QMF banks(7).

$$H_0(z)H_1(-z)-H_1(z)H_0(-z)=2Az^{-L}$$
$$F_0(z)=H_1(-z)$$
$$F_1(z)=-H_0(-z) \qquad (2)$$

where $A$ is an arbitrary constant. Then Eq. (1) is reduced to Eq. (3).

$$Y(z)=Az^{-L}X(z) \qquad (3)$$

Therefore, under the conditions of Eq. (2) it is obvious that Fig. 1 corresponds to a linear time invariant system, which has a simple time delay response.

Next, we consider the structure shown in Fig. 2, to discuss adaptive system identification using filter banks. In this case, we get the input-to-output relationship of this structure in the form

$$Y(z)=1/2\{H_0(z)A_0(z^2)F_0(z)+H_1(z)A_1(z^2)$$
$$\cdot F_1X(z)\}X(z)$$
$$+1/2\{H_0(-z)A_0(z^2)F_0(z)$$
$$+H_1(-z)A_1(z^2)F_1(z)\}X(-z). \qquad (4)$$

where $A_0(z)$ and $A_1(z)$ are transfer functions of adaptive filters.

Similarly, substituting Eq. (2) into Eq. (4), Eq. (4) may be rewritten as

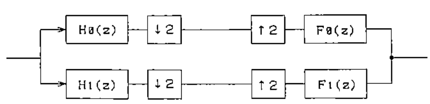


Fig. 1 a two-band filter bank.

Manuscript received July 19, 1991.
Manuscript revised August 26, 1991.

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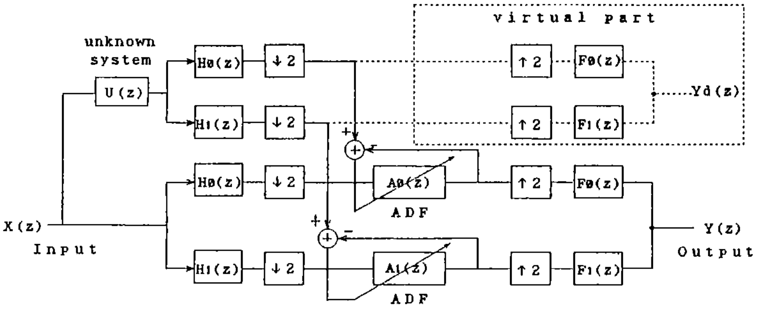



Fig. 2 Adaptive system identification based on a two-band filter bank.

$$Y(z)=1/2\{H_0(z)A_0(z^2)H_1(-z)-H_1(z)A_1(z^2)\cdot H_0(-z)\}X(z)+1/2H_0(-z)H_1(-z)\cdot\{A_0(z^2)-A_1(z^2)\}X(-z). \quad (5)$$

Unfortunately, the system in Fig. 2 does not correspond to a linear time invariant system, bacause the second part of the right side in Eq. (5) can not be cancelled, under Eq. (2).

By the way, giving Eq. (2), the output of the virtual part $Yd(z)$ in Fig. 2 is expressed as follows

$$Yd(z)=AU(z)z^{-L}X(z) \quad (6)$$

where $U(z)$ is a transfer function of an unknown system. We see from Eqs. (5) and (6) that it is impossible to express the unknown system $U(z)$ by using $A_0(z)$ and $A_1(z)$, even if the QMF bank is perfect.

## 3. Proposed Adaptive System Identification

In the previous section, we noted that the conditions for perfect QMF banks, namely, Eq. (2) do not correspond to those for system identification using filter banks. Thus, we also consider the conditions for filter banks, which enable us to identify any unknown system in this section.

One obvious solution for system identification using filter banks is to use the ideal filters. That is,

$$H_0(e^{jw})=F_0(e^{jw})=\begin{cases}1 & 0\leqq\omega<\pi/2\\ 0 & \pi/2\leqq\omega<\pi\end{cases} \quad (7)$$

$$H_1(e^{jw})=F_1(e^{jw})=\begin{cases}0 & 0\leqq\omega<\pi/2\\ 1 & \pi/2\leqq\omega<\pi\end{cases} \quad (8)$$

Using the above conditions, Eq. (4) may be modified in the form

$$Y(z)=1/2\{H_0^2(z)A_0(z^2)+H_1^2(z)A_1(z^2)\}X(z). \quad (9)$$

Obviously, the output of the virtual part $Yd(z)$ is the same as Eq. (6). Therefore we can identify any unknown system $U(z)$, from Eq. (9). With $A_0(z^2)$ and $A_1(z^2)$, the unknown system $U(z)$ is expressed as follows.

$$U(z)=\begin{cases}(1/2)A_0(z^2), & 0\leqq\omega\leqq\pi/2\\ (1/2)A_1(z^2), & \pi/2\leqq\omega\leqq\pi\end{cases} \quad (10)$$

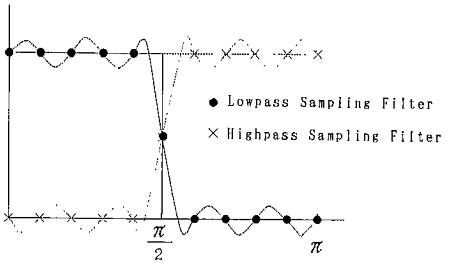


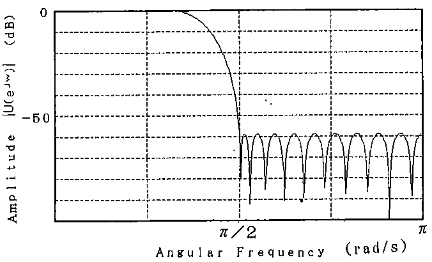

Fig. 3 Frequency sampling filter.

In practice, however, we cannot realize the ideal filters. So we propose a method with the frequency sampling filters shown in Fig. 3. That is, their frequency responses have $N$ equally spaced points on the ideal filter responses. When these filters are used in filter bank, Eq. (9) would be met at the discrete frequency points ($\omega_k=2\pi k/N$, $k=0, 1, 2, \cdots$), and then we can identify the unknown system at the discrete frequency points. The number of the sampling points, $N$ may be chosen with the needed accuracy. Also, we may use the FFT to execute these filters, and we can expand the above method in the case of an N-band filter bank, easily.

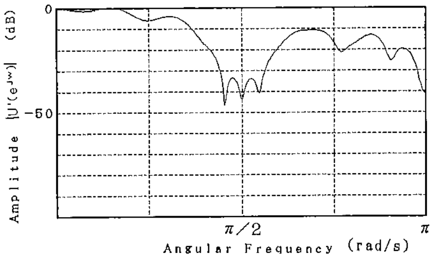
## 4. Simulation

We present some simulation examples of the adaptive system identification. In the examples, we assume the unknown system $U(z)$ having 33 taps shown in Fig. 4(a).

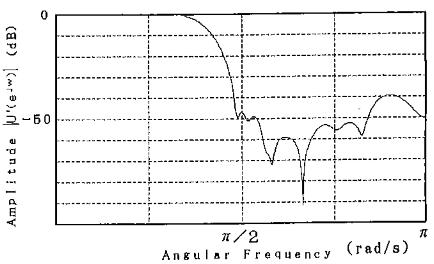
First, we use the two-band perfect QMF bank, referred to as SSKF (short symmetric Kernel filter)



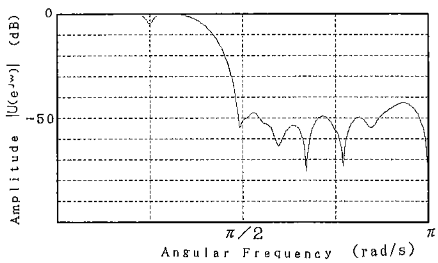

( a ) The unknown system $U(z)$.



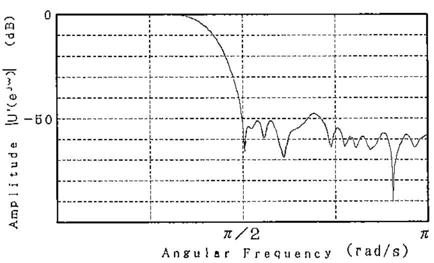

( b ) Two-band QMF bank composed of filters with 2 taps. (Length of adaptive filters is 17.)



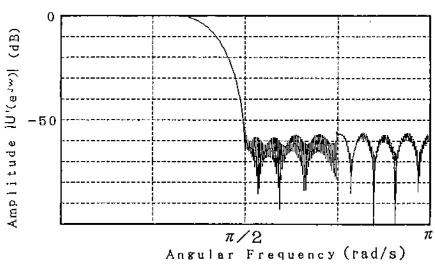

( c ) Two-band QMF bank composed of filters with 32 taps. (Length of adaptive filters is 17.)




( d ) Four-band QMF bank composed of filters with 32 taps. (Length of adaptive filters is 9.)




( e ) Four-band frequency sampling filter bank composed of filters with 32 taps. (Length of adaptive filters is 9.)




( f ) Four-band frequency sampling filter bank based on 512 points circular convolution. (length of adaptive filters is 9.)

Fig. 4 Frequency response of the identified functions.

bank[8], which is composed of four filters with 2 taps as follows,

$$H_0(z) = -F_1(-z) = 1 + z^{-1} \qquad (11)$$

$$H_1(z) = F_0(-z) = 1 - z^{-1}. \qquad (12)$$

These have poor frequency responses, although the bank is perfect. We have then identified the response shown in Fig. 4( b ) with the RLS algorithm, where the adaptive filters $A_0(z)$ and $A_1(z)$ have 17 taps, respectively. However this is obviously unsatisfactory. Also it was unsatisfactory, even in case of using adaptive filters having 33 taps. The reason is that this system does not correspond to a time invariant system, as shown in Eq. ( 5 ). Thus in this example the identified transfer function $U'(z)$ has been regarded as the right side of Eq. ( 4 ), except the second part, namely the aliasing part. That is,

$$U'(z) = 1/2\{H_0(z)A_0(z^2)F_0(z) + H_1(z)A_1(z^2)F_1(z)\} \qquad (13)$$

Next, we use the filters having 32 taps, for composing the two-band QMF bank reported in Ref. ( 9 ). Similarly, we have then obtained the transfer function $U'(z)$ as shown in Fig. 4( c ) from Eq. (13). Its performance is better than that of the previous example. This is because the responses of these filters are closer to those of the ideal filters, compared to the previous one. That is, if the bandstop attenuation in the frequency response is large, we may approximately neglect the aliasing component which the bandstop occurs in case of decimation process, as indicated in Eq. ( 5 ). However in general, we must choose a very high order filter to obtain such a response, especially if we want a filter having a narrow transient bandwidth.

Next we want to show that the influence of the aliasing component is emphasized when the number of subbands increases. Figure 4( d ) illustrates $U'(z)$ for the four-band QMF bank based on tree structures. Note that the error in the identified response mainly exists in neighbourhood of $\omega=\pi$ and $\omega=\pi/2$, due to the fact that the error is occurred by the aliasing component based on the transient band.

Figure 4( e ) shows that the proposed sampling filters with 32 taps are more effective than the example in Fig. 4( d ). Furthermore also we have used the sampling filters with 512 taps. Then we had the good result as shown in Fig. 4( f ). In this case, we used 512 points FFT to execute the circular convolution. Note that the frequency sampling filter is computationally efficient, even when used a very high order filter, if we may assume the circular convolution. This reason is that we do not need the product of the Fourier transforms and can directly do the process for the down sampling in the frequency domain.

## 5. Conclusion

In the letter, we have described that the conditions for parfect QMF banks do not correspond to those for system identification with filter banks, and that these conditions for system identification are to use the ideal filters. Next we have proposed a filter bank with sampling filters to improve the performance of the adaptive algorithm. This bank has some advantages, which are to identify an arbitrary unknown system at discrete frequency points, and are to be computationally efficient, even if we choose a high order filter, or a large number of band decomposition.

We will consider the computational complexity in more detail in the near future.

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