PAPER

A Linear Phase Two-Channel Filter Bank Allowing Perfect Reconstruction

Hitoshi KIYA†, Mitsuo YAE† *and* Masahiro IWAHASHI††, *Members*

SUMMARY We propose a design method for a two-channel perfect reconstruction FIR filter banks employing linear-phase filters. This type of filter bank is especially important in splitting image signals into frequency bands for subband image cording. Because in such an application, it is necessary to use the combination of linear-phase filters and symmetric image signal, namely linear phase signal to avoid the increase in image size caused by filtering. In this paper, first we summarize the design conditions for two-channel filter banks. Next, we show that the design problem is reduced to a very simple linear equation, by using a half-band filter as a lowpass filter. Also the proposed method is available to lead filters with fewer complexity, which enable us to use simple arithmetic operations. For subband coding, the property is important because it reduces hardware complexity.
key words: *digital filter, multirate digital signal processing, filter bank, subband coding*

1. Introduction

The purpose of this paper is to propose a design method for a two-channel filter bank using linear phase filters. This type of filter bank is especially important in splitting image signals into frequency bands for subband image cording. Because in such an application, it is necessary to use the combination of linear phase filters and symmetric image signal, namely linear phase signal to avoid the increase in image size caused by filtering, as noted in Refs. (1)-(4).

Work on filter banks was initiated by the introduction of the QMF(Quadrature Mirror Filter) concept.[(5),(6)] This two-channel QMF band which may be composed of linear phase filters can cancel the aliasing perfectly, while not solving the problem perfectly. The first perfect FIR solution was proposed by Smith and Barnwell.[(7)] However their solution does not include linear phase filters, that is, it must be composed of the minimum and maximum phase filters.

A perfect linear FIR solution for two-channel banks is respectively outlined in Refs. (8), (9) and (12). However, the length of the linear-phase filter shown in Ref. (8) is confined to 2, 3, 4 or 5. Also even if the method suggested in Ref. (9) is used, it is difficult to design filters having good response, such as the chebyshev approximation. The reason is that the procedure is executed without considering frequency responses. Compared to the above methods,[(8),(9)] the method suggested in Ref. (12) yields better filters in the practical sense. However when using this method, the computational complexity for designing filter banks is increased, due to the fact that this procedure is based on non-linear optimization. This can not lead filters with fewer complexity, which can be implemented by a limited number of shifts and adds and thus are computationally efficient, such as the SSKF (Symmetric Short Kernel Filter) bank.[(8)] For subband image coding, this type of filter is sometimes important because it reduces the hardware complexity.

Therefore in this paper, a new method for designing two-channel QMF banks is proposed. The proposed method has properties as follows: (a) it gives a linear phase filter bank allowing perfect reconstruction, (b) it is executed based on linear equations, (c) it can constrain frequency response for obtaining good amplitude response, (d) it may leads with fewer complexity, such as filters using simple coefficients i.e. ± 1 and 0, powers of 2, or simple integer. Any design method of the filter banks which have all of the above properties have never been presented.

2. Design Procedure

Figure 1 shows a block diagram of a two-channel filter bank. Then we obtain the input-to-output relationship of this system in the form[(9)]

$$Y(z)=\frac{1}{2}[H_0(z)F_0(z)+H_1(z)F_1(z)]X(z)$$
$$+[H_0(-z)F_0(z)+H_1(-z)F_1(z)]X(-z) \quad (1)$$

Suppose that Eq. (1) meets the following conditions, referred to as the necessary and sufficient condition for two-channel perfect QMF banks, namely perfect reconstruction

$$H_0(z)H_1(-z)-H_1(z)H_0(-z)=2z^{-L} \quad (2)$$

$$F_0(z)=H_1(-z) \quad (3)$$

Manuscript received March 13, 1992.
Manuscript revised October 15, 1992.
† The authors are with the Faculty of Technology, Tokyo Metropolitan University, Hachioji-shi, 192-03 Japan.
†† The author is with Nippon Steel Corporation, Sagamihara-shi, 229 Japan.

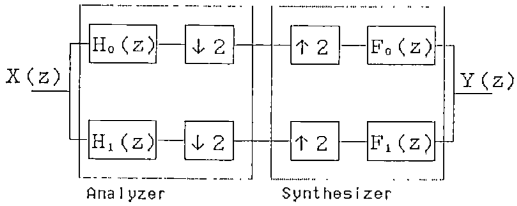


Fig. 1 Two-channel filter bank.

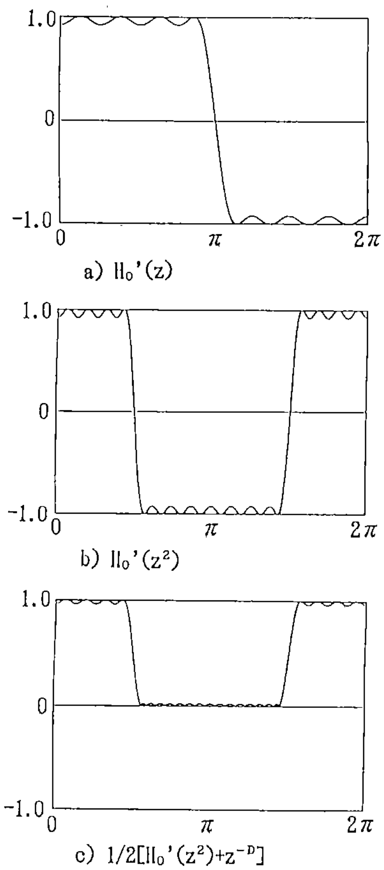


Fig. 2 A design procedure for linear-phase half band filters.

$$F_1(z) = -H_0(-z) \tag{4}$$

where L is an integer. Then Eq. (1) is reduced to Eq. (5).

$$Y(z) = z^{-L} X(z) \tag{5}$$

Therefore,our purpose is to show how to design $H_0(z)$ and $H_1(z)$ which have linear phases and satisfy Eq. (2).

The proposed method is summarized as follows.

Step 1: We design a half-band lowpass filter $H_0(z)$ (see Fig. 2).

1-1: Design a linear phase lowpass filter $H_0'(z)$ satisfying the following conditions:

* order D: Odd number
* passband edge ω_p: Arbitrary
* stopband edge ω_s: π

1-2: Modify $H_0'(z)$ in the form

$$H_0(z) = 1/2[H_0'(z^2) + z^{-D}] \tag{6}$$

Step 2: We choose the initial solution $H_1'(z)$ for designing a high pass filter as

$$H_1'(z) = 1 \tag{7}$$

Substituting (6) and (7) into (2), it is easily shown that Eq. (7) meets the condition for perfect reconstruction. That is, we obtain

$$H_0(z) H_1'(-z) - H_1'(z) H_0(-z) = z^{-D} \tag{8}$$

Step 3: We determine a high pass filter $H_1(z)$ in the form

$$H_1(z) = H_1'(z) z^{-J} - f(z) H_0(z) \tag{9}$$

where $f(z)$ and J satisfy the three conditions shown as;

1. $H_1(z)$ is a perfect solution if

$$J\text{: Even number} \tag{10}$$

$$f(z) = f(-z) \tag{11}$$

2. Under Eqs. (10), (11), $H_1(z)$ has a linear phase if

$$J = (2D + 2K)/2 \tag{12}$$

$$K\text{: Arbitrary odd number} \tag{13}$$

where $2K$ is the order of $f(z)$ having a linear phase.

3. Under Eqs. (10)-(13), $H_1(z)$ is a high pass filter if

$$f(z) H_0(z)\text{: Low pass filter having unit gain} \tag{14}$$

Now, we will explane why the above three conditions are needed, respectively. Condition 1. may be explained as follows, as noted in Ref. (10). By replacing $H_1(z)$ by $H_1'(z)$ in Eq. (2), we obtain

$$H_0(z) H_1'(-z) - H_0(-z) H_1'(z) = z^{-L} \tag{15}$$

Next multiplying (15) by z^{-J},

$$H_0(z) H_1'(-z)(-z)^{-J} - H_0(-z) H_1'(z) z^{-J} = z^{-(J+L)} \tag{16}$$

where J is an even number. By introduction of a function $f(z) = f(-z)$, the above equation is rewritten as

$$H_0(z)[H_1'(-z)(-z)^{-J} - f(-z) H_0(-z)] - H_0(-z)[H_1'(z) z^{-J} - f(z) H_0(z)] = z^{-(J+L)} \tag{17}$$

or equivalently

$$H_0(z) H_1(-z) - H_0(-z) H_1(z) = z^{-(J+L)} \tag{18}$$

where

$$H_1(z) = H_1'(z)z^{-J} - f(z)H_0(z) \quad (9)$$

Therefore, $H_1(z)$ in Eq. (9) will be a perfect solution if $H_1'(z)$ is a perfect solution.

Note that the $f(z)H_0(z)$ has $2D+2K+1$ impluse responses under condition 2, and the center of symmetry of them occureds at the $J=(2D+2K)/2$th sequence. Thus the $H_1(z)=z^{-J}-f(z)H_0(z)$ shown in Eq. (9) has symmetric impulse responses. Equation (9) also lead to a hipass filter if the $f(z)H_0(z)$ is a lowpass filter, because of $H_1(z)+f(z)H_0(z)=z^{-J}$.

3. Relation between $H_0(z)$ and $H_1(z)$

3.1 Influence of $f(z)$

Now, let's discuss the frequency response of the low pass filter $H_0(z)$ and the high pass filter $H_1(z)$ in more detail. That is, we are interested in how the frequency response of the $H_1(z)$ is decided form $H_0(z)$ and the $f(z)$ noted in Eq. (9).

As shown in Sect. 2, if we want $H_1(z)$ is a high pass filter, $f(z)H_0(z)$ should be chosen as a low pass filter. This means that the $f(z)$ should have a low pass characteristics, because of the relation of $H_1(z)+f(z)H_0(z)=z^{-J}$. Next, we should note that the $f(z)$ must satisfy Eqs. (11) and (16) to obtain perfect QMF banks. Also Eq. (11) corresponds that the odd-numbered impulse responses of $f(z)$ is zero, as $f(z)=a+cz^{-2}+az^{-4}$, and thus it is necessary to lead $f(z^{1/2})$ with odd order from Eq. (16), as $f(z^{1/2})=a+cz^{-1}+az^{-2}$, for deciding $f(z)$. As a result, we should choose the $f(z^{1/2})$ which is a low pass filter with wide pass band, as shown in the later examples.

3.2 Filter Bank with Fewer Complexity

Next, let's consider to lead filter banks with fewer complexity, such its filter coefficients, 1, 0, and −1, or powers of 2, or simple integers which enable us to use simple arithmethic operations. Many researchers have discussed a design method for such a filter and its amplitude constrain.(14)–(18)

Thus we can use the design techniques and transfer functions for low pass filters $H_0(z)$ and $f(z)$. It is noted that the conventional design methods for filter banks can not lead a high pass filter $H_1(z)$ with fewer complexity, because the $H_1(z)$ is decided by using numerical methods, even if we use $H_0(z)$ with fewer complexity. The other hand, using $H_0(z)$ and $f(z)$ with fewer complexity, our method can lead such a $H_1(z)$ from the simple relation noted in Eq. (9), as shown in Sect. 4.

Table 1 Impulse response of $H_0(z)$ (example 1). note: $h(n)=h(30-n)$

n	impulse response
0	−0.002098684332295
1	0.000000000000000
2	0.006155293134961
3	0.000000000000000
4	−0.014370627126239
5	0.000000000000000
6	0.028980876132764
7	0.000000000000000
8	−0.053894237350643
9	0.000000000000000
10	0.098046541538130
11	0.000000000000000
12	−0.193399659315670
13	0.000000000000000
14	0.630131582906360
15	0.999102171174737

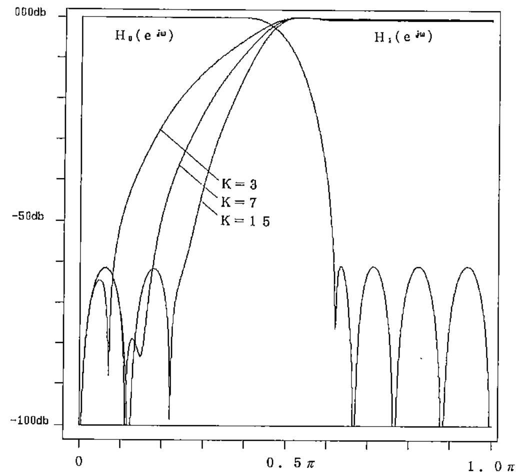


Fig. 3 Frequency responses (example 1). K: order of $f(z^{1/2})$

4. Design Examples and Consideration

4.1 Filter Banks with Multipliers

Example 1: We show a typical design example. First, we design a half-band low pass filter $H_0(z)$, as noted in Sect. 2. The filter coefficients in Table 1, which have a half-band characteristics are designed by using Remez algorithm (see Fig. 3). Next, in order to lead a high pass filter $H_1(z)$, we must determine $f(z)$, namely $f(z^{1/2})$. As discussed in Sect. 3, $f(z^{1/2})$ with odd order K should be a wide-band low pass filter. Therefore for example, we may choose $f(z^{1/2})$ from Table 2 which gives linear phase maximally flat FIR functions with odd order. Figure 4 shows the frequencies of these

Table 2 Linear-phase maximally flat functions with odd order. $f(z^{1/2})$.
note: $h(n)=h(K-n)$, K: order of $f(z^{1/2})$

n	K = 3	K = 5	K = 7
0	$-6.2500146917979\times10^{-2}$	$1.1718680702309\times10^{-2}$	$-2.4414527110208\times10^{-3}$
1	0.5624999257297	$-9.7656398268428\times10^{-2}$	$2.3925717063599\times10^{-2}$
2		0.5859374170170	-0.1196290534547
3			0.5981444440123

n	K = 9	K = 15
0	$5.3401343141035\times10^{-4}$	$-6.4095624961802\times10^{-6}$
1	$-6.1798585421572\times10^{-3}$	$1.1061853815747\times10^{-4}$
2	$3.4606872143076\times10^{-2}$	$-9.1532677863918\times10^{-4}$
3	-0.1345826653138	$4.8476788861765\times10^{-3}$
4	0.6056212482428	$-1.8698403022615\times10^{-2}$
5		$5.7590858009221\times10^{-2}$
6		-0.1599749110378
7		0.6170454438427

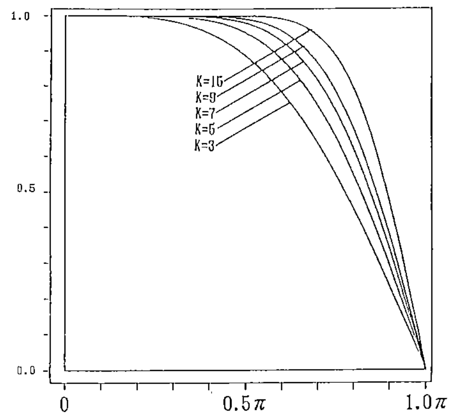


Fig. 4 Frequency responses of the maximally flat functions. K: order of $f(z^{1/2})$

filters, respectively.

Then, the $H_1(z)$ obtained from Eq. (9) is shown in Fig. 3 for each $f(z^{1/2})$. We see that $f(z)$ with higher order gives a good amplitude response. In order to obtain a good amplitude response for $H_1(z)$, it is necessary to use a $H_1(z)$ with higher order than $H_0(z)$, as noted in Ref. (12). It should be noted that we may choose other $f(z)$, such as the one designed by using Remez algorithm, or some window functions.

4.2 Filter Banks with Fewer Complexity

Example 2: Using this method, we lead a filter bank having simple filter coefficients. For example, as a half-band low pass filter $H_0(z)$, we may choose the simplest half-band filter as

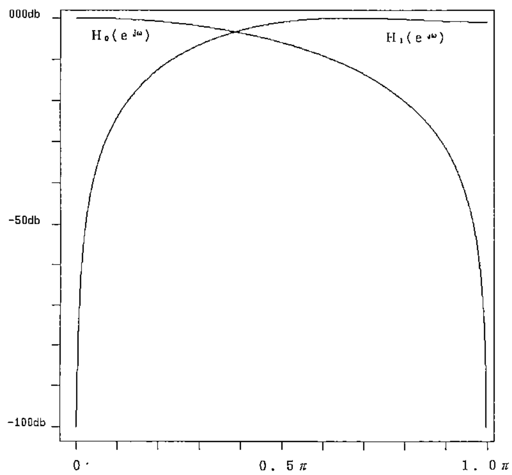


Fig. 5 Frequency responses (example 2).

$$H_0(z)=1/4(1+2z^{-1}+z^{-2}) \quad (19)$$

If we want to lead a high pass filter $H_1(z)$ with simple coefficients, we should use a simple $f(z)$ as

$$f(z)=1/2(1+z^{-2}) \quad (20)$$

Then, from Eq. (9), we get

$$H_1(z)=-1/8(1+2z^{-1}-6z^{-2}+2z^{-3}+z^{-4}) \quad (21)$$

Figure 5 shows the frequency responses of $H_0(z)$ and $H_1(z)$, respectively.

Example 3: It is well known that there are many transfer functions with simple coefficients.(14)–(18) Thus let show another design example under such a filter. For example, one of them is given by

Table 3 Approximate maximally flat functions with simple coefficients.
note: $h(n)=h(K-n)$, K: order of $f\,(z^{1/2})$

n	K = 3	K = 5	K = 7	K = 9	K = 1 5
0	-2^{-4}	2^{-7}	-2^{-9}	2^{-11}	-2^{-18}
1	2^{-1}	-2^{-4}	2^{-6}	-2^{-8}	2^{-14}
2		2^{-1}	-2^{-4}	2^{-5}	-2^{-11}
3			2^{-1}	-2^{-3}	2^{-8}
4				2^{-1}	-2^{-6}
5					2^{-5}
6					-2^{-3}
7					2^{-1}

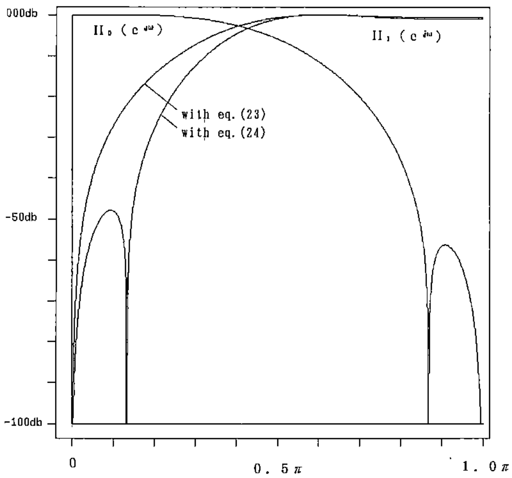


Fig. 6 Frequency responses (example 3).

$$H_0(z)=1/32(-1+8z^{-2}+14z^{-3}+8z^{-4}+z^{-6}) \quad (22)$$

where Eq. (22) is led from Table 3.

Next, we may choose $f(z)$ as

$$f(z)=1/2(1+z^{-2}) \quad (23)$$

or

$$f(z)=1/16(-1+8z^{-2}+8z^{-4}+z^{-6}) \quad (24)$$

where Eq. (24) chosen from Table 3 have maximally flat or approximate maximally flat characteristics, while the filter coefficients are very simple. Then, we get $H_1(z)$ as

$$H_1(z)=z^{-J}-f(z)H_0(z) \quad (25)$$

It is noted that the $H_1(z)$ consists of the filters with simple coefficients. Figure 6 shows the frequences of the filters.

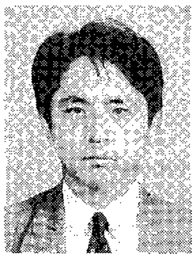
5. Concluding Remarks

We have described a perfect-reconstruction two-channel QMF bank, in which the analysis and synthesis filters have linear phases. First, a technique for designing filter banks has been presented. Next the results of the examples have demonstrated that its procedure is very simple, and also this can yields filters with simple coefficients such as the SSKF bank.

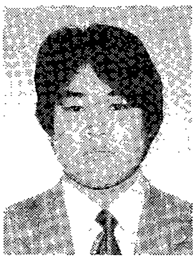
References

(1) Smith, M. J. T. and Eddins, S. L., "Analysis/Synthesis techniques for subband image coding," *IEEE Trans. Acoust., Speech & Signal Process.*, vol. 38, no. 8, Aug. 1990.
(2) Kim, C. -S., Bruder, J., Smith, M. J. T. and Mersereau, R. M., "Subband coding of color images using finite state vector quantization," *IEEE ICASSP 88*, Apr. 1988.
(3) Smith, M. J. T. and Eddins, S. L., "Subband coding images with octave band tree structures," *IEEE ICASSP 87*, Apr. 1987.
(4) Nishikawa, K., Kiya, H. and Sagawa, M., "Property of circular convolution for subband image coding," *IEEE ICASSP 92*, Mar. 1992.
(5) Esteban, D. and Galand, G., "Application of quadrature mirror filters to splitband voice coding schemes," *IEEE ICASSP 77*, May 1977.
(6) Croisier, A., Esteban, D. and Galand, G., "Perfect channel splitting by use of interpolation/decimation/tree decomposition techniques," *Int. Conf. on Information Science and Systems*, 1976.
(7) Smith, M. J. T. and Barnwell, T. P., "Exact reconstruction techniques for tree-structured subband coders," *IEEE Trans. Acoust., Speech & Signal Process.*, vol. 34, Jun. 1986.
(8) Le Gall, D. and Tabatabai, A., "Subband coding of digital images using symmetric short kernel filters and arithmetic coding techniques," *IEEE ICASSP 88*, 1988.
(9) Vetterli, M., "Filter banks allowing perfect reconstruction," *Signal Processing 10*, 1986.
(10) Kurosawa, K. and Yamamoto, K., "A design method of maximally decimated linear phase perfect QMF systems," *IEICE Paper of Technical Group on Circuit and Systems*, CAS91-9, May 1991.
(11) McClellan, J. H., Parks, T. W. and Rabiner, L. R., "A Computation Program for Designing Optimum FIR Linear Phase Digital Filters," *IEEE Trans. Audio and Electroacoust.*, vol. AU-21, no. 6, 1973.
(12) Nguyen, T. Q. and Vaidyanathan, P. P., "Two-channel perfect-reconstruction FIR QMF structures which yield linear-phase analysis and synthesis filters," *IEEE Trans. Acoust., Speech & Signal Process.*, vol. 37, May 1989.
(13) Aikawa, N., Yabiku, N. and Sato, M., "Design method of maximally flat FIR filter in consideration of even and odd order," *IEEE ISCAS 91*, Jun. 1991.
(14) Benvenuto, N., Franks, L. E. and Hill, jr F. S., "Realization of Finite Impulse Response Filters Using Coefficients +1, 0, and −1," *IEEE Trans. Commun.*, vol. 33, Nov. 1985.
(15) Bateman, M. R. and Liu, B., "An Approach to Programmable CTD Filters Using Coefficients 0, +1, and −1," *IEEE Trans. Circuits & Syst.*, vol. 27, Jun. 1980.
(16) Conboy, L. W. and Kaufmann, J., "Wideband General

Purpose FIR Filter Using Powers-of-Two Techniques," *IEEE ISCAS 86*, pp. 225–226, 1986.
(17) Xue, P. and Liu, B., "Adaptive Equalizer Using Finit-Bit Powe-of-Two Quantizer," *IEEE ISCAS 79*, pp. 185–188, 1979.
(18) Lueder, E., "Design and Optimization of Digital Filters Without Multipliers," *IEEE ISCAS 83*, pp. 1336–1339, 1983.



Hitoshi Kiya was born in Yamagata, Japan on November 16, 1957. He received the B.E. and M.E. degrees in electrical engineering from Nagaoka University of Technology, and the D.E. degree in electrical engineering from Tokyo Metropolitan University in 1980, 1982 and 1987, respectively. From 1982 to 1993, he was an Assistant Proffessor at Tokyo Metropolitan University, and he is currently an Associate Proffessor in Department of Electronics and Information Engineering at Tokyo Metropolitan University. His current reserch interests include multirate digital processing, image processing and adaptive signal processing. He is a co-author of four books which are "The Fast Fourier Transform Algorithm," "Digital Control System Analysis and Design," "The Fast Fourier Transform and Its Applications", and "Introduction to Digital Signal Processing." He is a member of IEEE CAS and ASSP Society, and a member of the institute of image electronics engineers of Japan.



Mitsuo Yae was born in Tokyo, Japan on September 24, 1967. He received the B.E. degree in electrical engineering from Tokyo Metropolitan University in 1991. He is currently a student of master cource. His reserch interests include the digital image processing.

Masahiro Iwahashi was born in Tokyo, Japan on February 24, 1965. He received the B.E. and the M.E. in electrical engineering from Tokyo Metropolitan University in 1988 and 1990 respectively. Since 1990 he has been with electronics laboratories of Nippon Steel Corporation, Kanagawa, Japan and worked on the development of digital image coding.