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A New Factorization Technique for the Generalized Linear-Phase LOT and Its Fast Implementation*

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SUMMARY In this work, a new structure of M-channel linear-phase paraunitary filter banks is proposed, where M is even. Our proposed structure can be regarded as a modification of the conventional generalized linear-phase lapped orthogonal transforms (GenLOT) based on the discrete cosine transform (DCT). The main purpose of this work is to overcome the limitation of the conventional DCT-based GenLOT, and improve the performance of the fast implementation. It is shown that our proposed fast GenLOT is superior to that of the conventional technique in terms of the coding gain. This work also provides a recursive initialization design procedure so as to avoid insignificant local-minimum solutions in the non-linear optimization processes. In order to verify the significance of our proposed method, several design examples are given. Furthermore, it is shown that the fast implementation can be used to construct M-band linear-phase orthonormal wavelets with regularity.

key words: *multirate filter banks, paraunitary system, linear-phase filter, subband image coding*

1. Introduction

In the area of audio and visual communications, the applications of multirate filter banks to data compression have been studied as an effective coding scheme, known as the subband coding (SBC) technique [1]. In the SBC applications, paraunitary (PU) property [2] of filter banks is of interest since it allows us to use optimal bit-allocation algorithms [3]. Besides, linear-phase (LP) property of each filter in the system is desired in image coding applications, since filter banks with LP property can handle finite-duration sequences without size-increase problem [4]–[8]. Hence, linear-phase paraunitary filter banks (LPPUFB) are particularly expected to be applied for SBC of images.

Several LPPUFB have been studied so far [9]–[16]. By Princen and Bradley in [9], and by Malver in [10], [11], special cases of such systems, of which polyphase matrices [2] are of order one, were shown, and their efficient implementation was established. The system developed in [10] is known as the lapped orthogonal transforms (LOT). By Vetterli et al. in [12] and by Soman et al. in [13], the more general systems of higher order were addressed, but the fast implementation was not considered.

Recently, in [14], Queiroz et al. constructed the generalized LP LOT (GenLOT) based on the type-II discrete cosine transform (DCT) [17], and investigated the fast implementation. The DCT-based GenLOT was, however, considered under some limitation, and this fact affects the achievable performance such as coding gain and stopband attenuation. In order to solve the limitation problem, the general form of GenLOT was provided in [15], [16] by replacing the DCT by another LP orthonormal matrix. However, the choice of the initial parameters in the design processes is more complicated than that of the DCT-based one, and the computational load of the newly introduced matrix is, in general, heavier than that of the fast DCT.

In this paper, to overcome the limitation problem in the DCT-based GenLOT, we propose a new structure of that and consider the fast implementation. The outline of this paper is as follows. In Sect. 2, we review LP and PU properties of M-channel maximally decimated filter banks, and in Sect. 3, we provide an overlap-save method (OLS) based on the type-II DCT (DCT-II) for FIR filtering. Then, in Sect. 4, we consider applying the OLS to factorize some class of LPPUFB. The results can be regarded as a new representation of the DCT-based GenLOT. In Sect. 5, we provide a recursive initialization design procedure to avoid insignificant local-minimum solutions, and establish the fast implementation by simplifying the structure as was done in [10], [11]. In order to verify the significance of our proposed method, several design examples and the computational complexity are shown.

2. Linear-Phase Paraunitary Filter Banks

As a preliminary, we review the M-channel maximally decimated filter banks, and also the PU [2] and LP properties [12], [13]. All through this work, the following notations are used.

$\boldsymbol{O}$: the null matrix.

$\boldsymbol{I}_M$: the $M \times M$ identity matrix.

$\boldsymbol{J}_M$: the $M \times M$ reversal (or counter-identity) matrix.

$\boldsymbol{\Gamma}_M$: the $M \times M$ diagonal matrix which has $+1$ and -1 elements alternatively on the diagonal, defined by $\boldsymbol{\Gamma}_M = \text{diag}\,[1, -1, \ldots, (-1)^{M-1}]$.

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$\boldsymbol{P}_M$: the $M \times M$ permutation matrix which permutes the even rows into the top half and the odd rows into the bottom half. For example,

$$\boldsymbol{P}_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1}$$

Besides, the superscript '$*$' denotes complex conjugation, and the superscripts 'T' and '$\dagger$' on a vector or matrix denote the transposition and hermitian transposition, respectively. Furthermore, the tilde notation '$\tilde{}$' over vector or matrix denotes the paraconjugation [2], for example, $\tilde{\boldsymbol{E}}(z) = \boldsymbol{E}^\dagger(1/z^*)$.

2.1 M-Channel Maximally Decimated Filter Banks

Figure 1 (a) shows a parallel structure of M-channel maximally decimated filter banks [2], where $H_k(z)$ and $F_k(z)$ are the analysis and synthesis filters, respectively. The boxes including $\downarrow M$ and $\uparrow M$ denote the downsampler and upsampler with the factor M, respectively. When the reconstructed output sequence $\hat{X}(z)$ is identical to the input $X(z)$, except for the delay and scaling, the analysis-synthesis system is called perfect reconstruction (PR) filter banks.

The structure as shown in Fig. 1 (a) can always be rewritten in terms of the polyphase matrices as shown in Fig. 1 (b), where $\boldsymbol{E}(z)$ and $\boldsymbol{R}(z)$ denote the $M \times M$ polyphase matrices [2] corresponding to analysis and synthesis banks, respectively. Let $\boldsymbol{h}(z)$ and $\boldsymbol{f}(z)$ be the $M \times 1$ column vectors defined by $\boldsymbol{h}(z) = [H_0(z), H_1(z), \cdots, H_{M-1}(z)]^T$ and $\boldsymbol{f}(z) = [F_0(z), F_1(z), \cdots, F_{M-1}(z)]^T$, respectively, and let $\boldsymbol{d}(z) = [1, z^{-1}, \cdots, z^{-(M-1)}]^T$. In terms of the polyphase matrices $\boldsymbol{E}(z)$ and $\boldsymbol{R}(z)$, $\boldsymbol{h}(z)$ and $\boldsymbol{f}(z)$ are respectively represented as $\boldsymbol{h}(z) = \boldsymbol{E}(z^M)\boldsymbol{d}(z)$ and $\boldsymbol{f}^T(z) = z^{-(M-1)}\tilde{\boldsymbol{d}}(z)\boldsymbol{R}(z^M)$. If $\boldsymbol{E}(z)$ and $\boldsymbol{R}(z)$ satisfy the following condition [2]:

$$\boldsymbol{R}(z)\boldsymbol{E}(z) = z^{-N}\boldsymbol{I}_M \tag{2}$$

for some integer N, then the system has PR property.

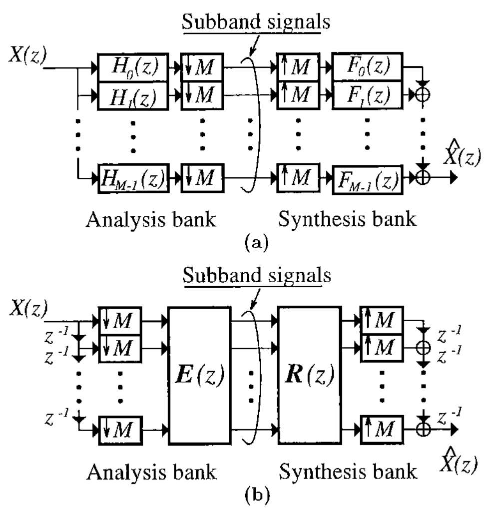


Fig. 1 M-channel maximally decimated filter banks. (a) and (b) show a parallel structure and the polyphase matrix representation, respectively.

2.2 Paraunitary (PU) Property

If $\boldsymbol{E}(z)$ satisfies the following condition [2]:

$$\tilde{\boldsymbol{E}}(z)\boldsymbol{E}(z) = \boldsymbol{I}_M, \tag{3}$$

then it is said to be paraunitary (PU).

The condition as in Eq. (3) is sufficient to construct PR filter banks, since the PR property as in Eq. (2) is guaranteed by choosing the synthesis polyphase matrix as $\boldsymbol{R}(z) = z^{-N}\tilde{\boldsymbol{E}}(z)$. When $\boldsymbol{E}(z)$ is causal FIR of order N, so is $\boldsymbol{R}(z)$ in this choice. Besides, it is of interest that the property as in Eq. (3) allows us to use optimal bit-allocation algorithms in the SBC applications [3].

2.3 Linear-Phase (LP) Property

Assume that $\boldsymbol{E}(z)$ is real and causal FIR of order N. On this assumption, the corresponding analysis filters $H_k(z)$ are also causal FIR with real coefficients and of order $K = (N+1)M - 1$. If $\boldsymbol{E}(z)$ further satisfies the following property [12], [13]:

$$z^{-N}\boldsymbol{\Gamma}_M\boldsymbol{E}(z^{-1})\boldsymbol{J}_M = \boldsymbol{E}(z), \tag{4}$$

then each analysis filter $H_k(z)$ for even k is symmetric and one for odd k is antisymmetric with the center of symmetry $K/2$. When the number of channel M is even, the analysis bank $\boldsymbol{h}(z)$ consists of $M/2$ symmetric and $M/2$ antisymmetric LP filters. Hence, the system described in Eq. (4) satisfies the necessary condition for LP PR filter banks with respect to the numbers of symmetric and antisymmetric filters [13, Theorem 1].

The complete factorization of LPPUFB as in Eqs. (3) and (4) has already been established in the articles [13], [15], [16] for even M. Our proposed factorization is also complete for the same class. Besides, the corresponding cascade structure is based on the DCT-II. As a result, a new representation of the DCT-based GenLOT is obtained and the fast implementation holding high coding gain can be established.

3. Overlap-Save Method Based on DCT

In this section, we provide an FIR filtering technique based on the DCT-II. The technique can be regarded as a modification of the generalized overlap-save method (OLS) [1] and has an important role for factorizing LP-PUFB described in Eqs. (3) and (4).

Let $H(z)$ be an FIR filter and $\boldsymbol{e}(z)$ be the $M \times 1$ vector defined by

$$e(z) = [E_0(z), E_1(z), \cdots, E_{M-1}(z)]^T, \tag{5}$$

where $E_\ell(z)$ is the ℓ-th type-I polyphase component of $H(z)$ with the decomposition factor M. In terms of $e(z)$, $H(z)$ can be represented as $H(z) = e^T(z^M)d(z)$. In the followings, we assume that the factor M is even.

In order to establish OLS with the DCT-II for FIR filtering, we firstly decompose $e(z)$ into the symmetric vector $s(z)$ and antisymmetric vector $a(z)$, as follows:

$$e(z) = s(z) + a(z) \tag{6}$$

where

$$s(z) = \frac{e(z) + J_M e(z)}{2}, \tag{7}$$

$$a(z) = \frac{e(z) - J_M e(z)}{2}. \tag{8}$$

Note that $s(z)$ and $a(z)$ are uniquely determined from their own $M/2 \times 1$ bottom-half vectors, which consist of their representative elements, respectively. By denoting the bottom-half vectors of $s(z)$ and $a(z)$ as $s^{\mathrm{r}}(z)$ and $a^{\mathrm{r}}(z)$, respectively, $e(z)$ can be represented as follows:

$$e^T(z) = \left[s^{\mathrm{r}T}(z) \;\; a^{\mathrm{r}T}(z) \right] \begin{bmatrix} J_{\frac{M}{2}} & I_{\frac{M}{2}} \\ -J_{\frac{M}{2}} & I_{\frac{M}{2}} \end{bmatrix}. \tag{9}$$

Then, we define transform coefficient vectors $g_{\mathrm{E}}(z)$ and $g_{\mathrm{O}}(z)$ of $s^{\mathrm{r}}(z)$ and $a^{\mathrm{r}}(z)$, respectively, by

$$g_{\mathrm{E}}(z) = \Gamma_{\frac{M}{2}} C^{\mathrm{II}}_{\frac{M}{2}} s^{\mathrm{r}}(z), \tag{10}$$

$$g_{\mathrm{O}}(z) = \Gamma_{\frac{M}{2}} S^{\mathrm{IV}}_{\frac{M}{2}} a^{\mathrm{r}}(z), \tag{11}$$

where C^{II}_M and S^{IV}_M denote the M-point orthonormal DCT-II and type-IV discrete sine transform (DST) matrices [17], respectively (see Appendix). Substituted the relations $s^{\mathrm{r}T}(z) = g_{\mathrm{E}}{}^T(z)\Gamma_{\frac{M}{2}} C^{\mathrm{II}}_{\frac{M}{2}}$ and $a^{\mathrm{r}T}(z) = g_{\mathrm{O}}{}^T(z)\Gamma_{\frac{M}{2}} S^{\mathrm{IV}}_{\frac{M}{2}}$, Eq. (9) can be rewritten as

$$e^T(z) = \sqrt{2}\left[g^T_{\mathrm{E}}(z) \;\; g^T_{\mathrm{O}}(z) \right] P_M C^{\mathrm{II}}_M J_M, \tag{12}$$

where the properties $P_M P^T_M = I_M$, $\Gamma_M C^{\mathrm{II}}_M = C^{\mathrm{II}}_M J_M$ and $\Gamma_M S^{\mathrm{IV}}_M = C^{\mathrm{IV}}_M J_M$ and the sparse matrix factorization of the DCT-II [17]

$$C^{\mathrm{II}}_M = \frac{1}{\sqrt{2}} P^T_M \begin{bmatrix} C^{\mathrm{II}}_{\frac{M}{2}} & O \\ O & C^{\mathrm{IV}}_{\frac{M}{2}} \end{bmatrix} \begin{bmatrix} I_{\frac{M}{2}} & J_{\frac{M}{2}} \\ I_{\frac{M}{2}} & -J_{\frac{M}{2}} \end{bmatrix} \tag{13}$$

are used, where C^{IV}_M is the M-point orthonormal type-IV DCT matrix [17]. From Eq. (12), an equivalent structure to $H(z)$ can be obtained as shown in Fig. 2. The structure can be regarded as a special case of the generalized OLS [1] using the DCT-domain filtering technique [18].

Assume that the order of the polyphase component vector $e(z)$ is N. In this case, the order of $H(z)$ results in $K = (N+1)M - 1$. Note that if and only if $H(z)$ is symmetric with the center of symmetry $K/2$, that is, the case that $z^{-N} e^T(z^{-1}) J_M = e^T(z)$, then the following properties are satisfied with $\gamma_{\mathrm{E}} = 1$ and $\gamma_{\mathrm{O}} = -1$:

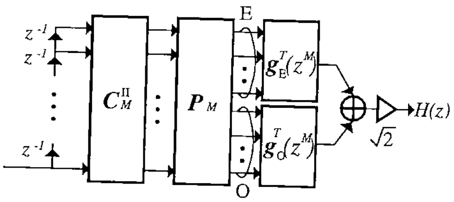


Fig. 2 A structure of DCT-based OLS for FIR filtering. The letters 'E' and 'O' represent even and odd coefficients, respectively.

$$g_{\mathrm{E}}(z) = \gamma_{\mathrm{E}} z^{-N} g_{\mathrm{E}}(z^{-1}), \tag{14}$$

$$g_{\mathrm{O}}(z) = \gamma_{\mathrm{O}} z^{-N} g_{\mathrm{O}}(z^{-1}). \tag{15}$$

Besides, if and only if $H(z)$ is antisymmetric with the center of symmetry $K/2$, that is, the case that $-z^{-N} e^T(z^{-1}) J_M = e^T(z)$, the above properties are satisfied with $\gamma_{\mathrm{E}} = -1$ and $\gamma_{\mathrm{O}} = 1$. These properties can be used to factorize LPPUFB as we will show in the next section.

4. New Structure of DCT-Based GenLOT

In this section, by using the DCT-based OLS developed in the previous section, we discuss a factorization technique of LPPUFB satisfying Eqs. (3) and (4) for even M. Our proposed factorization provides a new structure of the DCT-based GenLOT [14], which covers the same class as that of the general form [15], [16].

Assume that $E(z)$ is causal FIR of order N and satisfies the condition as in Eq. (4), and that the number of channels M is even. As mentioned before, on this assumption, the corresponding analysis filters $H_k(z)$ are causal FIR of order $K = (N+1)M - 1$, and the analysis bank $h(z)$ consists of $M/2$ symmetric and $M/2$ antisymmetric LP filters.

Let $e_k(z)$ be the type-I polyphase component vector of $H_k(z)$ provided as in Eq. (5), that is, the transpose of the k-th row vector of $E(z)$. Since $e_k(z)$ can be represented with the DCT-II as in Eq. (12) and satisfies the LP properties Eqs. (14) and (15), $E(z)$ can be rewritten as the following form:

$$E(z) = P^T_M G(z) P_M C^{\mathrm{II}}_M J_M, \tag{16}$$

where $G(z)$ is the $M \times M$ matrix which consists of the transform coefficient vectors obtained from $e_k(z)$ as in Eqs. (10) and (11), and has the form

$$G(z) = \sqrt{2} \begin{bmatrix} G_{\mathrm{ES}}(z) & G_{\mathrm{OA}}(z) \\ G_{\mathrm{EA}}(z) & G_{\mathrm{OS}}(z) \end{bmatrix}. \tag{17}$$

In Eq. (17), $G_{\mathrm{ES}}(z)$, $G_{\mathrm{OA}}(z)$, $G_{\mathrm{EA}}(z)$ and $G_{\mathrm{OS}}(z)$ denote $M/2 \times M/2$ matrices of order N which satisfy the properties

$$G_{-\mathrm{S}}(z) = z^{-N} G_{-\mathrm{S}}(z^{-1}), \tag{18}$$

$$G_{-\mathrm{A}}(z) = -z^{-N} G_{-\mathrm{A}}(z^{-1}), \tag{19}$$

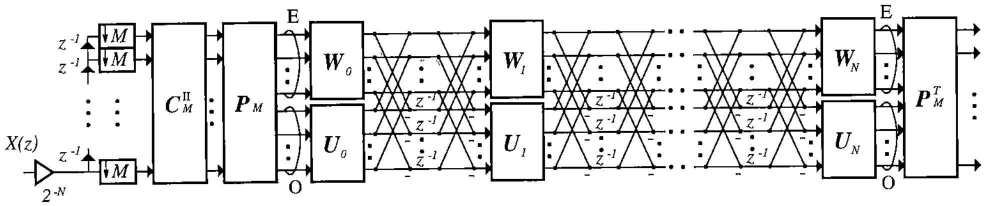


Fig. 3 A new cascade structure of the DCT-based GenLOT, where M denotes the number of channels and is even, and besides N denotes the order of the corresponding polyphase matrix $\boldsymbol{E}(z)$. The letters 'E' and 'O' represent even and odd coefficients, respectively.

where the subscript '-' stands for either 'E' or 'O'. The top half sub-matrix of $\boldsymbol{G}(z)$ corresponds to symmetric filters and the rest does antisymmetric ones.

Then, let us consider factorizing $\boldsymbol{G}(z)$ satisfying the property Eq. (17) under the PU constraint Eq. (3). Note that if and only if $\boldsymbol{E}(z)$ is PU, the $\boldsymbol{G}(z)$ is PU since all of $\boldsymbol{P}_M$, $\boldsymbol{J}_M$ and $\boldsymbol{C}_M^{\mathrm{II}}$ are PU. For convenience of the further discussion, we define the $M \times M$ matrix $\boldsymbol{F}(z)$ by $\boldsymbol{F}(z) = \boldsymbol{TBG}(z)$ where

$$\boldsymbol{T} = \begin{bmatrix} \boldsymbol{I}_{\frac{M}{2}} & \boldsymbol{O} \\ \boldsymbol{O} & \boldsymbol{J}_{\frac{M}{2}} \end{bmatrix}, \ \boldsymbol{B} = \frac{1}{\sqrt{2}} \begin{bmatrix} \boldsymbol{I}_{\frac{M}{2}} & \boldsymbol{I}_{\frac{M}{2}} \\ \boldsymbol{I}_{\frac{M}{2}} & -\boldsymbol{I}_{\frac{M}{2}} \end{bmatrix}.$$

Note that both of $\boldsymbol{T}$ and $\boldsymbol{B}$ are orthonormal. The matrix $\boldsymbol{F}(z)$ can be represented as

$$\boldsymbol{F}(z) = \boldsymbol{T} \begin{bmatrix} \boldsymbol{F}_{\mathrm{E}}(z) & \boldsymbol{F}_{\mathrm{O}}(z) \\ z^{-N}\boldsymbol{F}_{\mathrm{E}}(z^{-1}) & -z^{-N}\boldsymbol{F}_{\mathrm{O}}(z^{-1}) \end{bmatrix}, \quad (20)$$

where $\boldsymbol{F}_{\mathrm{E}}(z) = \boldsymbol{G}_{\mathrm{ES}}(z) + \boldsymbol{G}_{\mathrm{EA}}(z)$ and $\boldsymbol{F}_{\mathrm{O}}(z) = \boldsymbol{G}_{\mathrm{OS}}(z) + \boldsymbol{G}_{\mathrm{OA}}(z)$. From Eq. (20), it can be verified that $\boldsymbol{F}(z)$ satisfies the following property:

$$z^{-N}\boldsymbol{J}_M\boldsymbol{F}(z^{-1}) \begin{bmatrix} \boldsymbol{I}_{\frac{M}{2}} & \boldsymbol{O} \\ \boldsymbol{O} & -\boldsymbol{I}_{\frac{M}{2}} \end{bmatrix} = \boldsymbol{F}(z). \quad (21)$$

Once the above relation was obtained, as done in the proof for [13, Theorem 3], any $\boldsymbol{F}(z)$ can completely be factorized under the PU constraint Eq. (3) as

$$\boldsymbol{F}(z) = \boldsymbol{TBR}_N\boldsymbol{BA}(z)\boldsymbol{BR}_{N-1}\boldsymbol{B}\cdots\boldsymbol{A}(z)\boldsymbol{BR}_0, \quad (22)$$

where

$$\boldsymbol{R}_m = \begin{bmatrix} \boldsymbol{W}_m & \boldsymbol{O} \\ \boldsymbol{O} & \boldsymbol{U}_m \end{bmatrix}, \boldsymbol{\Lambda}(z) = \begin{bmatrix} \boldsymbol{I}_{\frac{M}{2}} & \boldsymbol{O} \\ \boldsymbol{O} & z^{-1}\boldsymbol{I}_{\frac{M}{2}} \end{bmatrix}.$$

In the above equation, $\boldsymbol{W}_m$ and $\boldsymbol{U}_m$ are $M/2 \times M/2$ orthonormal matrices. It should be noted that $\boldsymbol{F}(z)$ of which order is zero has the form $\boldsymbol{F}(z) = \boldsymbol{TBR}_0$. Substituted the relation $\boldsymbol{G}(z) = \boldsymbol{BTF}(z)$ and Eq. (22), Eq. (16) can be represented as follows:

$$\boldsymbol{E}(z) = \boldsymbol{P}_M^T\boldsymbol{R}_N\boldsymbol{Q}(z)\boldsymbol{R}_{N-1}\cdots\boldsymbol{Q}(z)\boldsymbol{R}_0\boldsymbol{P}_M\boldsymbol{C}_M^{\mathrm{II}}\boldsymbol{J}_M, \quad (23)$$

where $\boldsymbol{Q}(z) = \boldsymbol{B\Lambda}(z)\boldsymbol{B}$, which is also PU.

From Eq. (23), we notice that any PU analysis bank described in Eq. (4) for even M can always be constructed with the cascade structure as shown in Fig. 3, where the scaling factors $1/\sqrt{2}$ involved in $\boldsymbol{B}$ are unified, so that the result is 2^{-N}. Conversely, we can utilize the structure to design LPPUFB by controlling $\boldsymbol{W}_m$ and $\boldsymbol{U}_m$. Because of the PU property of $\boldsymbol{E}(z)$, the counterpart synthesis bank $\boldsymbol{R}(z)$ holding PR property is simply obtained as $\boldsymbol{R}(z) = z^{-N}\tilde{\boldsymbol{E}}(z)$.

From Fig. 3, the structure can be regarded as a new representation of the DCT-based GenLOT [14]. The conventional DCT-based GenLOT is viewed as the special case that $\boldsymbol{R}_0 = \boldsymbol{I}_M$. Note that the limitation of $\boldsymbol{R}_0$ affects the achievable performance such as coding gain and stopband attenuation. Hence, the performance of the fast implementation based on the conventional technique is also limited. The proposed GenLOT can overcome this problem and allows us to construct the fast implementation holding high coding gain or stopband attenuation, nevertheless it is based on the DCT-II.

5. Design Procedure and Fast Implementation

In the previous section, we developed a factorization technique of LPPUFB. In the followings, the design procedure is discussed, and then the fast implementation is established so as to reduce both of the design and implementation complexities. The proposed fast algorithm can achieve higher coding gain or stopband attenuation than the conventional one. In order to verify the significance of our proposed method, several design examples are also shown. Besides, we give some comments on the regularity, which is important to construct M-band wavelets [13], [15].

5.1 Recursive Initialization Design Procedure

According to the factorization as in Eq. (23), we can construct any M-channel LPPUFB satisfying Eqs. (3) and (4) for even M by controlling $2(N+1)$ $M/2 \times M/2$ orthonormal matrices $\boldsymbol{W}_m$ and $\boldsymbol{U}_m$ in the structure as shown in Fig. 3. Since each $\boldsymbol{W}_m$ and $\boldsymbol{U}_m$ can completely be characterized in terms of $M(M-2)/8$ Givens rotations (or planar rotations) [1], [2], it is allowed to design such a system by means of an unconstrained optimization process to minimize (or maximize) some object function. Both of the PU property as in Eq. (3) and the LP property as in Eq. (4) are guaranteed while designing since these constraints are structurally imposed.

In the optimization process, the initial parameters have to be carefully chosen so as to avoid insignificant local-minimum solutions. In both of the proposed and conventional GenLOT, the problem can almost be solved by recursively initializing the parameters to be optimized. Let $\boldsymbol{E}_m(z)$ be a matrix of order m represented as in Eq. (23). It can be verified that, when

$$\boldsymbol{W}_m = \boldsymbol{W}_{m-1} = \boldsymbol{I}_{\frac{M}{2}}, \quad (24)$$

$$\boldsymbol{U}_m = \boldsymbol{U}_{m-1} = -\boldsymbol{I}_{\frac{M}{2}}, \quad (25)$$

there exists a matrix $\boldsymbol{E}_{m-2}(z)$, and the matrix $\boldsymbol{E}_m(z)$ can be represented as follows:

$$\boldsymbol{E}_m(z) = z^{-1}\boldsymbol{E}_{m-2}(z). \quad (26)$$

Equation (26) implies that $\boldsymbol{E}_m(z)$ is identical to the two lower order system $\boldsymbol{E}_{m-2}(z)$ but with the delay. Hence, when $\boldsymbol{E}_{m-2}(z)$ has good performance, for example high coding gain, so does $\boldsymbol{E}_m(z)$. As a result, we can design a significant GenLOT for minimizing some object function by starting with a simple problem, and evolutionary increasing the parameters as the following procedure:

Step 1: Start with proper LPPUFB $\boldsymbol{E}_0(z)$ for even N or $\boldsymbol{E}_1(z)$ for odd N, for example DCT-II or LOT, and optimize it.

Step 2: Initialize the two higher order system by adding two sections according to Eq. (23) with the matrices as in Eqs. (24) and (25).

Step 3: Optimize the system, and go to **Step 2** until the order reaches to N.

Note that the starting guess in Step 1 is easily achieved, since the proposed structure is based on the DCT-II, and that the above procedure does not depend on the choice of object function. Although it does not guarantee the global minimum solution, experimental results show that it leads to a significant solution.

In the entire structure as in Eq. (23), the number of free parameters, that is, rotation angles, to be optimized is $(N+1)(M-2)M/4$, and the implementation requires $\mu(\boldsymbol{C}_M^{\rm II}) + (N+1)M^2/2$ multiplications and $\alpha(\boldsymbol{C}_M^{\rm II}) + (N+1)(M-2)M/2 + 2NM$ additions per block, where $\mu(\boldsymbol{C}_M^{\rm II})$ and $\alpha(\boldsymbol{C}_M^{\rm II})$ denote the number of multiplications and additions of M-point DCT-II, respectively, and it is assumed that each of $\boldsymbol{W}_m$ and $\boldsymbol{U}_m$ requires $M^2/4$ multiplications and $(M-2)M/4$ additions. As well known, DCT-II has the fast implementations [17], and therefore, can be efficiently implemented.

Indeed, the proposed GenLOT is slightly inefficient compared with the conventional general form. However, since the DCT-II is a good approximation to the optimum solution of the first transform matrix in the general form, the complexity of the proposed structure is considerably reduced by some simplification, holding good performance.

5.2 Fast Implementation

In order to reduce both of the design and implementation complexities of the proposed GenLOT, we consider simplifying the matrices $\boldsymbol{W}_m$ and $\boldsymbol{U}_m$ as

$$\boldsymbol{W}_m = \boldsymbol{I}_{\frac{M}{2}}, \quad (27)$$

$$\boldsymbol{U}_m = \boldsymbol{T}_{m,\frac{M}{2}-2}\boldsymbol{T}_{m,\frac{M}{2}-3}\cdots\boldsymbol{T}_{m,0} \quad (28)$$

for $m = 0, 1, \cdots, N$, respectively, in the similar way to the type-I fast LOT [10], [11], where

$$\boldsymbol{T}_{m,i} = \begin{bmatrix} \boldsymbol{I}_i & \boldsymbol{O} & \boldsymbol{O} \\ \boldsymbol{O} & \boldsymbol{Y}(\theta_{m,i}) & \boldsymbol{O} \\ \boldsymbol{O} & \boldsymbol{O} & \boldsymbol{I}_{\frac{M}{2}-i-2} \end{bmatrix}, \quad (29)$$

$$\boldsymbol{Y}(\theta_{m,i}) = \begin{bmatrix} \cos\theta_{m,i} & -\sin\theta_{m,i} \\ \sin\theta_{m,i} & \cos\theta_{m,i} \end{bmatrix}. \quad (30)$$

By the above simplification, the number of rotation angles $\theta_{m,i}$ to be optimized is reduced to $(N+1)(M-2)/2$, and the implementation complexity is also reduced to $\mu(\boldsymbol{C}_M^{\rm II}) + 3(N+1)(M-2)/2$ multiplications and $\alpha(\boldsymbol{C}_M^{\rm II}) + 3(N+1)(M-2)/2 + 2NM$ additions per block, where it is assumed that each $\boldsymbol{U}_m$ requires $3(M-2)/2$ multiplications and $3(M-2)/2$ additions. As we will show experimentally, this simplification does not lead significant reduction of the coding gain. Note that the recursive initialization approach is still available.

5.3 Design Examples

In order to verify the significance of the proposed GenLOT and its fast implementation, we show some design examples, where the object function of optimization is chosen as the maximum coding gain $G_{\rm TC}$ [3].

Table 1 shows the resulting $G_{\rm TC}$'s of the proposed GenLOT and its fast structure which are optimized for an AR (1) signal with $\rho = 0.95$, and also their implementation complexities, where the number of channels

Table 1 Coding gain $G_{\rm TC}$ of several transforms, for an AR (1) signal with $\rho = 0.95$ and computational complexities ($M = 8$). N denotes the order of the corresponding polyphase matrix. #MUL's and #ADD's stand for the numbers of multiplications and additions per block, respectively.

TRANSFORM	N	$G_{\rm TC}$ [dB]	#MUL's [/Block]	#ADD's [/Block]
DCT-II	0	8.825	13	29
LOT-Fast I	1	9.198	22	54
Conventional DCT-based GenLOT	2	9.180	77	109
	3	9.360	109	149
Proposed GenLOT	0	8.846	45	53
	1	9.269	77	93
	2	9.394	109	133
	3	9.463	141	173
Proposed Fast GenLOT	0	8.827	22	38
	1	9.232	31	63
	2	9.315	40	88
	3	9.438	49	113

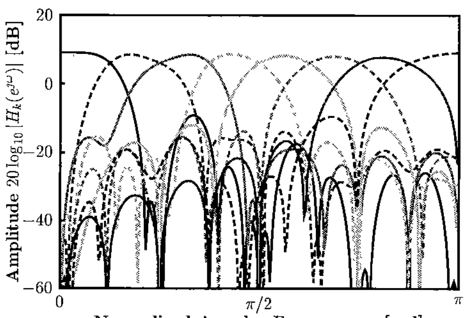
M was fixed to 8. Those of DCT-II [17], the type-I fast LOT (denoted as LOT- FAST I) [10], [11] and the conventional DCT-based GenLOT [14] are also shown, where any simplification for fast implementation is not assumed for the conventional GenLOT.

From Table 1, we notice the following things: 1) G_{TC} of the proposed fast GenLOT is comparable to that of the entire structure, and the implementation is more efficient. 2) G_{TC} of the proposed fast GenLOT is higher than that of the conventional DCT-based GenLOT where no simplification is assumed, and the implementation is more efficient. Summarizing, the fast implementation of the proposed fast GenLOT is superior to the conventional technique in terms of the coding gain, in spite of the parameter reduction.

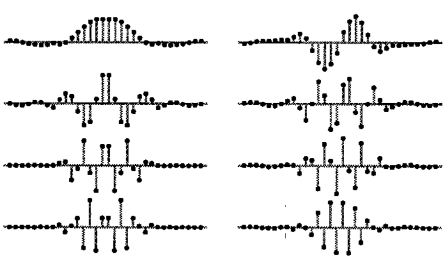
As an example, Table 2 gives the optimized angles $\theta_{m,i}$ of the proposed fast GenLOT, where $M = 8$ and $N = 3$. Besides, the amplitude and impulse responses of the optimized analysis filters $H_k(z)$ are given in Fig. 4.

Table 2 A design example of the proposed fast GenLOT: angles $\theta_{m,i}$ optimized for an AR (1) signal with $\rho = 0.95$ ($M = 8, N = 3$).

$\theta_{m,i}$		$i=0$	$i=1$	$i=2$
m	0	-0.15π	-0.02π	-0.04π
	1	1.29π	-0.03π	0.93π
	2	1.17π	-0.01π	1.05π
	3	0.85π	-0.15π	1.19π



(a)



(b)

Fig. 4 A design example of the proposed fast GenLOT for an AR (1) signal with $\rho = 0.95$ ($M = 8, N = 3, K = 31$). (a) and (b) show the amplitude and impulse responses of 8 analysis filters $H_k(z)$, respectively.

5.4 Regularity

The use of the GenLOT enables us to obtain M-band LP orthonormal wavelets by iterating the decomposition [13], [15]. The condition that the continuous time wavelets have at least one vanishing moment is that $\boldsymbol{h}(1) = \boldsymbol{E}(1)\boldsymbol{d}(1) = [\sqrt{M}, 0, 0, \cdots, 0]^T$. In this case, there is no DC leakage into the higher frequency subbands. It can be verified that, in the proposed GenLOT, this condition is satisfied when the product of the matrices $\boldsymbol{W}_m$ has the form

$$\boldsymbol{W}_N\boldsymbol{W}_{N-1}\cdots\boldsymbol{W}_0 = \begin{bmatrix} 1 & \boldsymbol{O} \\ \boldsymbol{O} & \boldsymbol{V} \end{bmatrix}, \tag{31}$$

where $\boldsymbol{V}$ is an $(M/2-1) \times (M/2-1)$ orthonormal matrix. The above condition is easily derived from the facts that $\boldsymbol{P}_M\boldsymbol{C}_M^{II}\boldsymbol{J}_M\boldsymbol{d}(1) = [\sqrt{M}, 0, 0, \cdots, 0]^T$ and $\boldsymbol{Q}(1) = \boldsymbol{I}_M$. Obviously, our proposed fast GenLOT satisfies the above condition and, therefore, is applicable to construct M-band wavelets with regularity.

6. Conclusion

In this work, we proposed a new structure of M-channel linear-phase paraunitary filter banks for even M, which can be regarded as a new representation of the conventional DCT-based GenLOT. The most significant feature of our proposed method is that the fast implementation is achievable by some simplification, holding high coding gain. The significance was verified by several design examples. It was also shown that the fast implementation is applicable to construct M-band linear-phase orthonormal wavelets with regularity. In future, we will provide the size-limited structure for the applications to subband image codings, and develop the discussion to construct multidimensional non-separable linear-phase perfect reconstruction filter banks.

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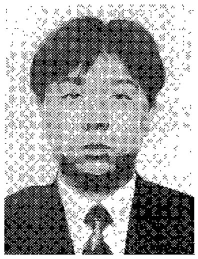
Appendix: Definitions of DCT-II and DST-IV

The M-point orthonormal type-II DCT and type-IV DST matrices are defined as follows, respectively [17]:

$$\left[C_M^{\rm II}\right]_{mn} = \sqrt{\frac{2}{M}} k_m \cos\left(\frac{m(n+\frac{1}{2})\pi}{M}\right), \quad (A\cdot 1)$$

$$\left[S_M^{\rm IV}\right]_{mn} = \sqrt{\frac{2}{M}} \sin\left(\frac{(m+\frac{1}{2})(n+\frac{1}{2})\pi}{M}\right), \quad (A\cdot 2)$$

for $0 \leq m, n \leq M-1$, where the notation $[\cdot]_{mn}$ denotes the mn-th element of its argument matrix, and k_m is $1/\sqrt{2}$ for $m = 0$ and 1 for $1 \leq m \leq M-1$.



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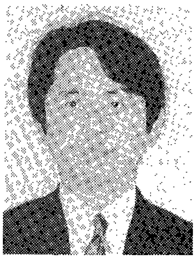

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