

# A Frequency Domain Adaptive Algorithm for Estimating Impulse Response with Flat Delay and Dispersive Response Region

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**SUMMARY** In some applications, such as the echo cancellation problem of satellite-linked communication channels, there occurs a problem of estimation of a long impulse response, which consists of a long flat delay and a short dispersive response region. In this paper, it is shown that the use of the adaptive algorithm based on the frequency domain sampling theorem enables efficient identification of the long impulse response. The use of the proposed technique can lead to the reduction of both the number of adaptive weights and the complexity of flat delay estimation. **key words:** *impulse response, delay, frequency domain, system identification, dispersive response region*

## 1. Introduction

In some adaptive filter applications, the unknown impulse response consists of several delays and short dispersive response regions [1]–[7]. In typical applications, such as echo path estimation of satellite-linked communication channels, the delay is as long as over 600 milliseconds, whereas the dispersive response region is only of the order of less than 10 milliseconds [7].

Figure 1 shows a model [5] of the impulse response of the echo path which has a long flat delay and a short dispersive response region. In the model,  $L$ ,  $D$  and  $N$  represent the length of the impulse response, the length of the flat delay and the length of the dispersive region, respectively. The flat delay arises from the large propagation delay in the communication channels, and it increases the tap length of the adaptive filters.

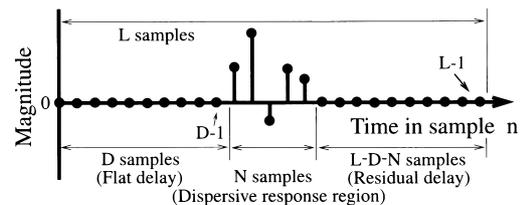
To estimate the impulse response with the flat delay and dispersive response region, several time domain adaptation algorithms have been studied. The STWQ (scrub taps waiting in a queue) [4] and the SELQUE (select and queue with a constraint) [5]–[7] algorithms enable efficient reduction of the number of adaptive coefficients by utilizing a sparse tap adaptive finite impulse response (FIR) filter, as shown in Fig. 2.

Although the STWQ and SELQUE algorithms de-

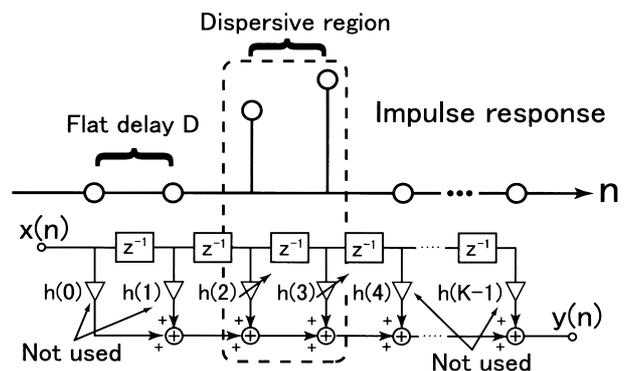
crease the number of adaptive coefficients, the delay estimation algorithm and the impulse response identification algorithm cannot be separated. If the flat delay changes, these algorithms must restart their tap allocation and adaptation processes from the initial state.

In this paper, we propose the use of the frequency domain adaptive algorithm [8] based on the frequency domain sampling theorem [9]. Since the proposed technique evaluates the subband signals in equally spaced discrete frequency points regardless of the length of the flat delay  $D$ , it can lead to the reduction of both the number of adaptive weights and the complexity of the flat delay estimation. In addition, the flat delay estimation and the impulse response estimation of the dispersive region can be performed independently.

In the following section, the problem is defined. Section 3 describes the proposed frequency domain algorithm. In Sect. 4, the impulse response identification algorithm of the dispersive region and flat delay esti-



**Fig. 1** A model of the impulse response with flat delay and dispersive region.



**Fig. 2** Conventional time domain adaptation scheme for the impulse response with flat delay  $D$ .

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mation problem are considered. Section 5 is a discussion of the arithmetic complexity. Finally, in Sect. 6, simulation results are provided, which confirm the effectiveness of the proposed method.

## 2. Problem Definition

Let us define the model of the unknown impulse response [5] with total length  $L$ , as shown in Fig. 1. The model consists of a flat delay of length  $D$ , a dispersive response region of length  $N$  and residual delay  $L - D - N$ . In the model,  $z^{-D}$  represents the flat delay and an actual delay  $T_D$ [sec] is given by  $T_D = D \times T_s$ [sec], where  $T_s$ [sec] represent sampling interval.

In typical applications, such as echo path estimation problems of satellite-linked communication channels, it is well known that  $L, D \gg N$ . The flat delay  $D$  and the total length  $L$  depend on the propagation delay in communication channels and the dispersive response region corresponds to the model of an echo from a hybrid.

In this paper, let us assume the following conditions.

- (a) The lengths  $L$  and  $N$  are unknown, while the upper bounds of  $L$  and  $N$  are given.
- (b)  $L > N$  is satisfied.
- (c) The length  $D$  is unknown *a priori*.
- (d) The position of the flat delay region and the dispersive region are unknown *a priori*.
- (e) The energy of the samples which corresponds to the flat delay region and the residual delay region is negligibly small.

## 3. Frequency Domain Adaptive Algorithm Based on the Frequency Domain Sampling Theorem

### 3.1 Frequency Domain Sampling Theorem

First, we consider the frequency domain sampling theorem [9]. For simplicity, we use the notation  $F(\omega)$  instead of the notation  $F(e^{j\omega})$ .

[Frequency domain sampling theorem] Let  $F(\omega)$  be the discrete time Fourier transform (DTFT) of a finite duration discrete sequence  $f(n)$ , ( $0 \leq n \leq N - 1$ ). We denote this relationship as

$$f(n) \stackrel{\text{DTFT}}{\leftrightarrow} F(\omega). \tag{1}$$

The frequency domain sample  $F(\omega_k)$  is defined as

$$F(\omega_k) = F(\omega)|_{\omega=\frac{2\pi k}{M}} \tag{2}$$

$$(k = 0, 1, \dots, M - 1).$$

If the condition  $M \geq N$  is satisfied, the discrete sequence  $f(n)$  can be perfectly determined from the inverse discrete Fourier transform (IDFT) of  $F(\omega_k)$ .

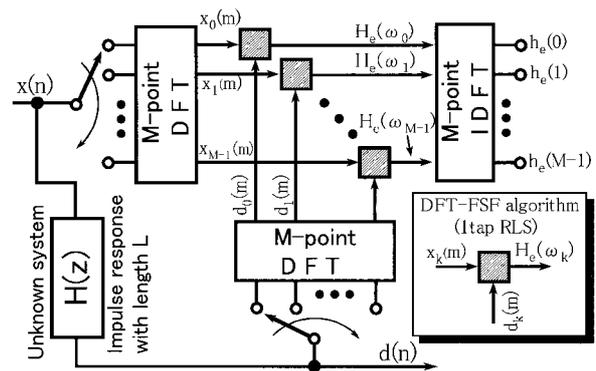


Fig. 3 Implementation of the frequency domain adaptive system identification block diagram utilizing the DFT-FSF banks.

Note that, in the above theorem,  $F(\omega)$  denotes a continuous spectrum, whereas  $F(\omega_k)$  denotes a line spectrum.

### 3.2 Frequency Domain Identification Utilizing the DFT-FSF Banks

Let us consider frequency domain system identification methods [8],[10]–[12], which utilize the DFT frequency sampling filter (DFT-FSF) banks. The DFT-FSF bank is a family of the alias-free frequency sampling filter (FSF) bank [13],[14] and these filter banks have alias-free properties at equally spaced discrete frequency points.

Figure 3 shows the  $M$  subband implementation of the adaptive system identification block diagram, based on the frequency domain sampling theorem, and Table 1 shows the DFT-FSF adaptive algorithm [8]. The limiting value analysis of the algorithm is considered in [11].

$H(z)$  denotes an unknown system and  $\mathbf{h}$  represents its impulse response of length  $L$ .  $H(\omega)$  is the frequency response of  $H(z)$ .  $x(n)$  and  $d(n)$  represent the input and desired signals, respectively. The unknown impulse response  $\mathbf{h}$  can be expressed as

$$\mathbf{h} = [h(0), h(1), \dots, h(L - 1)]. \tag{3}$$

$H(\omega_k)$ , ( $\omega_k = \frac{2\pi k}{M}, k = 0, 1, \dots, M - 1$ ) denotes the frequency domain samples of the unknown system, while  $H_e(\omega_k)$  denotes the estimation of  $H(\omega_k)$ . By applying the frequency domain identification algorithm, as shown in Table 1, we can obtain  $H_e(\omega_k)$ .

If the condition  $M \geq L$  is satisfied, the estimated unknown impulse response  $\mathbf{h}_e$  can be obtained from the IDFT of  $H_e(\omega_k)$  based on the frequency domain sampling theorem. The estimated unknown impulse response  $\mathbf{h}_e$  can be expressed as

$$\mathbf{h}_e = [h_e(0), h_e(1), \dots, h_e(M - 1)], \tag{4}$$

$$h_e(n) = \begin{cases} h(n) & 0 \leq n \leq L - 1 \\ 0 & L \leq n \leq M - 1. \end{cases} \tag{5}$$

**Table 1** DFT-FSF adaptive algorithm using the frequency domain 1-tap adaptive weights.

$m$ : time variable	$k$ : bank number
for $k = 0$ to $M - 1$	
$\hat{x}_k(m) = \alpha \hat{x}_k(m - 1) + x_k(m)$	
$\hat{d}_k(m) = \alpha \hat{d}_k(m - 1) + d_k(m)$	
$r_k(m) = \lambda r_k(m - 1) + \hat{x}_k(m) \hat{x}_k^*(m)$	
$p_k(m) = \lambda p_k(m - 1) + \hat{d}_k(m) \hat{x}_k^*(m)$	
$H_e(\omega_k) = p_k(m) / r_k(m)$	
* denotes complex conjugate.	

Hence the DFT-FSF adaptive system has an  $M$ -channel bin and 1-tap adaptive weight in each bin; it corresponds to the  $M$  tap adaptive system in the time domain [8].

**4. Impulse Response Identification of the Dispersive Region**

The proposed algorithm can separate the impulse response identification algorithm and the flat delay estimation algorithm. In this section, we describe the identification algorithm to identify the impulse response of the dispersive response region defined in Fig. 1.

Figure 4(a) represents the DTFT pair of the unknown impulse response  $h$  and its frequency domain representation  $H(\omega)$ . Figures 4(b) and (c) represent the impulse response  $h_e$  which is obtained by the IDFT of the estimated frequency domain samples  $H_e(\omega_k)$ . The estimated impulse response  $h_e(n)$  can be expressed as

$$h_e(n) = \frac{1}{M} \sum_{k=0}^{M-1} H_e(\omega_k) e^{j \frac{2\pi nk}{M}}, \quad (6)$$

$$(0 \leq n \leq M - 1).$$

Our goal is to estimate the impulse response of the dispersive response region using only  $M(M > N)$  adaptive weights in the frequency domain. Let us consider the following two cases of  $M \geq L$  and  $L > M \geq N$ , as shown in Figs. 4(b) and (c).

**4.1 Redundancy of the Estimated Impulse Response**

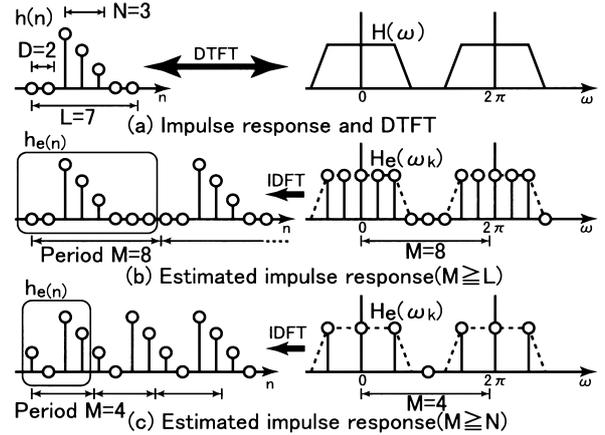
In this section, we assume that the condition  $M \geq L$  is satisfied. Figure 4(b) shows an example in the case of  $L = 7$  and  $M = 8$ . In this case, the frequency domain sampling theorem is satisfied. Therefore, from Eq. (6), we can obtain the entire unknown impulse response  $h_e$  as a discrete sequence with period  $M$  shown in Eq. (4).

In addition, the obtained impulse response  $h_e$  has redundant zeros, as shown in the next equation.

$$h_e(n) \simeq \begin{cases} 0 & 0 \leq n \leq D - 1 \\ 0 & D + N \leq n \leq M - 1 \end{cases} \quad (7a)$$

$$(7b)$$

Equation (7a) deals with the flat delay  $D$  and Eq. (7b) deals with the redundancy of the DFT of size  $M$ .



**Fig. 4** The time domain and the frequency domain representation of the estimated impulse response of the dispersive response region.

**4.2 Proposed Estimation Method**

In this section, we assume that the condition  $L > M \geq N$  is satisfied. Figure 4(c) shows an example for the case of  $L = 7$  and  $M = 4$ . In this case, for the condition  $M < L$ , the frequency domain sampling theorem is not satisfied. Usually, this leads to time domain aliasing in the obtained impulse response  $h_e$ . However, the obtained sequence  $h_e$  is not disturbed by the time domain aliasing.

The reason for this is as follows.

- (A) The proposed algorithm identify the unknown system at the alias-free discrete frequencies based on the frequency domain sampling theorem.
- (B) From the definition, the unknown impulse response has only  $N$  non zero samples and the other samples are negligibly small.
- (C)  $h_e$  is determined as the discrete sequence with period  $M$ , where  $M > N$  is satisfied.

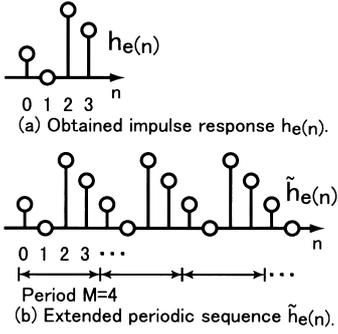
Since the time domain aliasing arises only in the region represented by Eqs. (7a) and (7b), the obtained sequence  $h_e$  not disturbed by the time domain aliasing.

However, due to the influence of the flat delay  $D$ , an extra phase shift term arises in the estimated frequency domain samples  $H_e(\omega_k)$ . As a result, the relationship in Eq. (5) is not satisfied.

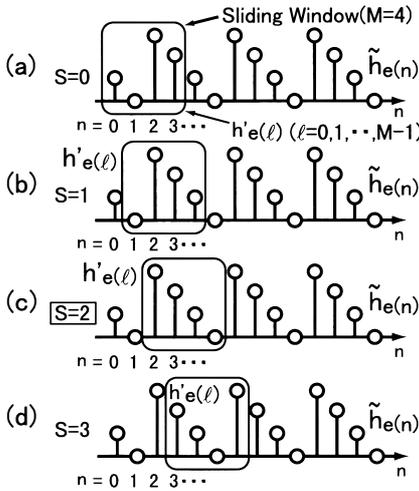
Let us consider the obtained impulse response  $h_e(n)$  and the extended version of the periodic sequence  $\tilde{h}_e(n)$ , as shown in Fig. 5. The relationship between  $h_e(n)$  and  $\tilde{h}_e(n)$  is described as

$$\tilde{h}_e(n) = \begin{cases} h_e(n) & 0 \leq n \leq M - 1 \\ \tilde{h}_e(n - M) & \text{otherwise.} \end{cases} \quad (8)$$

Let us denote the true estimated impulse response



**Fig. 5** The obtained impulse response  $h_e(n)$  and extended periodic sequence  $\tilde{h}_e(n)$ .



**Fig. 6** An example of the candidates of the true estimated impulse response  $h'_e(\ell)$ , ( $L = 7, D = 2, N = 3, M = 4$ ).

of the dispersive region as

$$\mathbf{h}'_e = [h'_e(0), h'_e(1), \dots, h'_e(M-1)]. \quad (9)$$

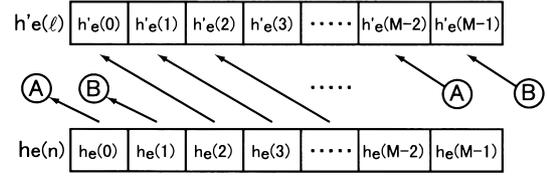
In Fig. 6, the candidates of the true estimated impulse response  $h'_e(\ell)$  ( $\ell = 0, 1, \dots, M-1$ ), which correspond to the dispersive region of the unknown impulse response, are depicted in the case of  $M = 4$ . Obviously, since  $\tilde{h}_e(n)$  is a periodic sequence with period  $M$ , there are only  $M$  independent candidates, as shown in Fig. 6 (a) to (d). The relationship between the candidates  $h'_e(\ell)$  and  $\tilde{h}_e(n)$  is described as

$$h'_e(\ell) = \tilde{h}_e(\ell + S), \quad (0 \leq \ell \leq M-1). \quad (10)$$

where  $S$  ( $0 \leq S \leq M-1$ ) is defined as the shift value.

On the other hand, from the conditions (A), (B) and (C) described in Sect. 4.2,  $\mathbf{h}'_e$  should have redundant nearly zero components in the tail. The length of zero components is  $M - N$ . Thus, the true estimated impulse response  $\mathbf{h}'_e$  should have the following form.

$$\mathbf{h}'_e = [h'_e(0), h'_e(1), \dots, h'_e(N-1), \underbrace{0, \dots, 0}_{M-N}] \quad (11)$$



**Fig. 7** The circular rotation representation of the obtained impulse response (in the case of  $S = 2$ ).

From Eqs. (10) and (11), we can choose the correct impulse response of the dispersive region by selection from the  $M$  candidates of  $\mathbf{h}'_e$ .

The above considerations are summarized as follows.

[Estimation procedure of  $\mathbf{h}'_e$ ]

- step1** To obtain  $\mathbf{h}_e$ , running the algorithm which described in Table 1 and calculate Eq. (6).
- step2** Define the periodic sequence  $\tilde{h}_e(n)$  as shown in Eq. (8).
- step3** By using of the sliding window with length  $M$ , as shown in Fig. 6, extract  $M$  candidates of  $\mathbf{h}'_e$  in the case of  $S = 0$  to  $S = M-1$ .
- step4** Choose a  $\mathbf{h}'_e$  which satisfies Eq. (11) from the selection of the above candidates.
- step5** Record the value of  $S$  which correspond to the selected candidate.  $S$  will be use an estimation of the flat delay.

In the above procedure, a condition  $M > N$  is required. From the definition, the length of  $N$  is unknown while the upper bound of  $N$  is given, therefore we can choose  $M$  to satisfy the necessary condition  $M > N$ . In Fig. 6 (c), an example of the selected candidate using the above procedure with  $S = 2$  in the case of  $M = 4, N = 3$  is given.

To represent the relationship between  $h_e(n)$  and  $h'_e(\ell)$ , let us consider a circular rotation representation of  $h_e(n)$  and  $h'_e(\ell)$  as shown in Fig. 7. From Eqs. (8) and (10), the desired impulse response  $h'_e(\ell)$  can be written as

$$h'_e(\ell) = \begin{cases} h_e(\ell + S) & 0 \leq \ell \leq M - S - 1 \\ h_e(\ell - M + S) & M - S \leq \ell \leq M - 1, \end{cases} \quad (12)$$

where

$$S = ( (D) )_M = (D_{\text{mod}M}). \quad (13)$$

In Eq. (13),  $(D_{\text{mod}M})$  represents an operation to take the remainder after division of  $M$ . Note that, although  $S$  is given by Eq. (13), the above estimation procedure of  $\mathbf{h}'_e$  should be used instead of Eq. (12) as  $D$  was assumed to be unknown.

The proposed technique only requires  $M$  ( $M > N$ ) adaptive weights in the frequency domain even in the case of the long impulse response with total length  $L$  and flat delay  $D$ .

### 4.3 Restriction of the Proposed Method

In the estimation procedure of  $h'_e$ , which described in the previous section, the restriction of the value of  $h'_e$  is not considered.

In Eq. (11),  $h'_e(\ell)(0 \leq \ell \leq N - 1)$  should not include the zero vector of order greater than  $M - N$ . If this condition is not satisfied, the **step3** is not available.

To avoid this problem, one of the following conditions is required.

- (a) Choose the DFT size as  $M > N + O_z$ .  $O_z$  represents the order of the zero vector in  $h'_e(\ell)(0 \leq \ell \leq N - 1)$ .
- (b) Choose the DFT size as  $M > 2N$ .

As a result, it increases an arithmetic complexity of the proposed method. However, the cost is not quite large under the condition  $L \gg N$ .

### 4.4 Estimation of the Flat Delay

From the previous discussion, the upper bound of  $D$  is given by  $L - M$ , while the flat delay  $D$  becomes a quantized value which is a multiple of  $M$ . From Eq. (13), the possible values of  $D$  can be expressed by

$$D = k \times M + S, \tag{14}$$

$$0 \leq k \leq \left\lceil \frac{L - M}{M} \right\rceil.$$

In Eq. (14),  $k$  represents an integer and the notation  $\lceil \cdot \rceil$  represents the smallest integer which is not less than its argument.

There are  $k$  possibilities for determining the flat delay  $D$ . The selection of  $D$  can be done from observations of the residual error  $e(n)$ , using a convolution

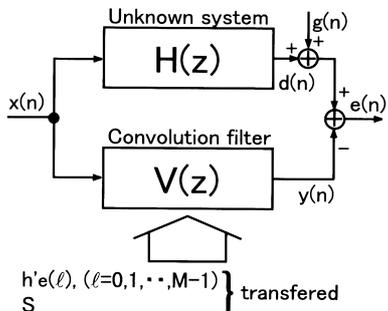


Fig. 8 Echo suppression scheme of the proposed method.

filter  $V(z)$  with length  $D + M$  as shown in Fig. 8. In Fig. 8,  $x(n)$  and  $y(n)$  represent the input signal and the echo replica, respectively.  $g(n)$  represents a model of additive noise. It is guaranteed that we can determine the flat delay  $D$  after the  $k$ -th trial.

## 5. Discussion of the Complexity of the Implementation

In this section, let us consider the complexity involved in the implementation of the proposed technique. Table 2 summarizes the proposed technique and the conventional time domain algorithm for the following four items.

- $L_{conv}$ : The length of the entire delay in the convolution filter.
- $C_{tap}$ : The number of combinations of tap allocations in the delay estimation problem after  $D$  has changed.
- $Q_{Len}$ : The length of the queue in the algorithm.
- $O_c$ : The order of the arithmetic complexity of the adaptive filtering per a sample.

In Table 2, we assume that the number of adaptive coefficients is  $M$ .

Let us consider the number of combinations  $C_{tap}$  after  $D$  has changed. In the proposed technique, the flat delay estimation and the impulse response estimation are performed independently, thus the impulse response estimation accuracy is not disturbed by the changing of the flat delay  $D$ . As a result, if  $D$  changes, according to Eq. (14)  $C_{tap}$  becomes  $\lceil \frac{L-M}{M} \rceil$  even in the worst case, as shown in Table 2.

In the STWQ and SELQUE, the flat delay estimation and the impulse response estimation algorithms are not separated. Thus, in this case, these algorithms should restart their flat delay estimation and tap update processes from the initial state even if only  $D$  has changed. In the STWQ and SELQUE,  $C_{tap}$  is given by  $L - N + 1$ , as shown in Table 2.

Figure 9 shows the comparison of the value of  $C_{tap}$  in the case of  $N = 0.1L$  and  $N = 0.01L$ . We can significantly reduce the value of  $C_{tap}$  under the condition  $L \gg N$  for large  $L$ .

In Table 2, the length  $Q_{Len}$  in the proposed algorithm represents the value of  $k$  in Eq. (14), while  $Q_{Len}$  for STWQ and SELQUE represents the total length of the waiting queue for active tap control. For the proposed method,  $O_c$  includes arithmetic complexity of the additional convolution filter as shown in Fig. 8.

Table 2 Complexity of implementation of algorithms.

	Proposed	STWQ [4]	SELQUE [6]
Length of delay: $L_{conv}$	$L$	$L$	$L$
Tap allocation: $C_{tap}$	$\lceil \frac{L-M}{M} \rceil$	$L - M + 1$	$L - M + 1$
Length of queue: $Q_{Len}$	$\lceil \frac{L-M}{M} \rceil$	$L - M$	$L - M$
Adaptive filtering: $O_c$	$O(\log_2 M + M)$ [8]	$2M$	$2M$

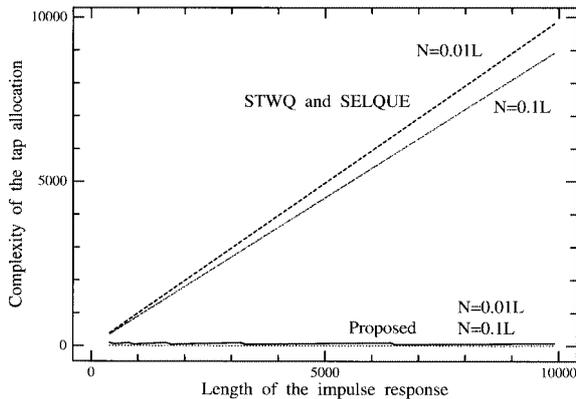


Fig. 9 Comparison of the complexity of tap re-allocation.

### 6. Simulation Example

In this section, by using the proposed method, the impulse response identification of the dispersive response region is demonstrated in a simulation example. Then, we will consider comparison of the mean square error (MSE) convergence property of the proposed method and the conventional time domain algorithm.

All simulation examples are done under the 32bit floating point operation.

#### 6.1 IRER Convergence

[Conditions]

- Unknown system:  $L = 53, D = 50, N = 3$ .
- Dispersive region:  $h(50) = 1, h(51) = 2, h(52) = -1$  (as shown in Fig. 10 (a)).
- DFT size:  $M = 4$ .
- Input signal: Gauss noise with variance  $\sigma^2 = 1$ , mean value  $m = 0$ .
- Forgetting constant:  $\alpha = \lambda = 0.999$ .

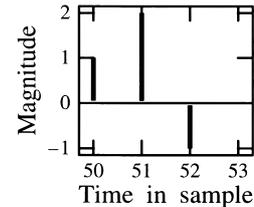
Figure 10 (a) shows the dispersive region of the unknown impulse response. Figures 10 (b) and (c) show the obtained impulse response  $h_e(n)$  and the impulse response estimation ratio (IRER) convergence, respectively. The definition of the IRER is given by

$$\text{IRER} = 10 \log_{10} \frac{|\mathbf{h} - \mathbf{h}'_e|^T |\mathbf{h} - \mathbf{h}'_e|}{\mathbf{h}^T \mathbf{h}} \quad (15)$$

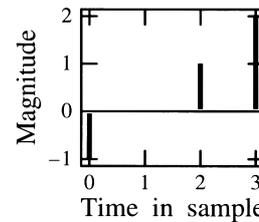
In this example, by using of the estimation procedure of  $\mathbf{h}'_e$ , we can obtain  $S = 2$ . The estimated impulse response  $h'_e(\ell)$  are as follows in the case of  $\text{IRER} = -49.0$  [dB].

$$\begin{aligned} h'_e(0) &= 9.997744560e - 01 \simeq h(50) \\ h'_e(1) &= 1.993155241e + 00 \simeq h(51) \\ h'_e(2) &= -9.947283268e - 01 \simeq h(52) \\ h'_e(3) &= 5.498528481e - 04 \simeq h(53) \end{aligned}$$

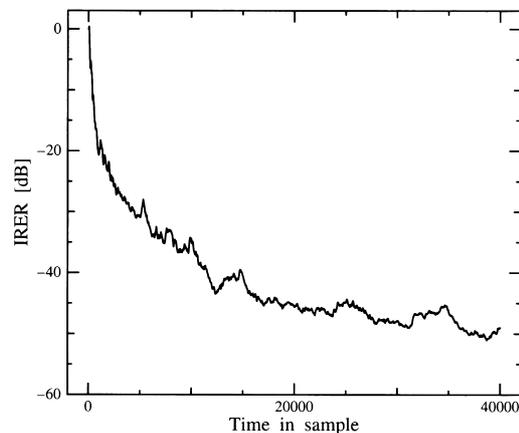
In addition, by substituting  $S = 2$  into Eq. (12), we



(a) Dispersive region of the unknown impulse response  $h(n)$  with the total length  $L = 53$ .



(b) Obtained impulse response  $h_e(n)$  ( $M = 4, S = 2$ ).



(c) IRER convergence characteristics.

Fig. 10 Simulation example of the proposed method ( $L = 53, D = 50, N = 3, M = 4$ ).

can confirm the relationship between  $h'_e(\ell)$  and  $h_e(n)$  as shown in Fig. 10 (b), where  $h'_e(\ell)$  is a estimated dispersive response region of  $h(n)$ .

By using the proposed technique, we can identify the impulse response of the dispersive region using only  $M$  adaptive weights in the frequency domain.

#### 6.2 Comparison of the LMS Algorithm

We consider the comparison of the least mean square (LMS) algorithm and the proposed method assuming an echo cancellation. Figure 8 shows an echo canceler block diagram.

The unknown impulse response  $h(n)$  is defined as

$$\begin{aligned} \text{Flat delay :} & \quad h(0) \text{ to } h(D - 1) \\ \text{Dispersive region :} & \quad h(D) \text{ to } h(D + N - 1) \\ \text{Residual delay :} & \quad h(D + N) \text{ to } h(L - 1), \end{aligned} \quad (16)$$

where  $L = 400, N = 51$  and  $D = 90, 100, 155$ .

The dispersive response region of the echo path  $h(k)$  is defined as [5]

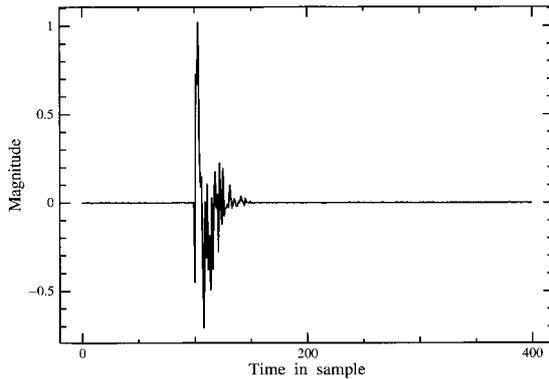


Fig. 11 Unknown impulse response ( $D = 100$ ).

$$h(k) = \exp\{-0.1 \times (k - D)\} \cdot w(k) \quad (17)$$

$$k = D, D + 1, \dots, D + N - 1.$$

where  $w(k)$  is Gaussian noise of zero mean with unit variance. The flat delay and residual delay region consist of Gaussian noise of zero mean with variance of  $10^{-6}$ . These Gaussian noise are uncorrelated with together. Figure 11 shows the unknown impulse response in the case of  $D = 100$ .

The input signal  $x(n)$  is 40000 points colored signal which generated by the coloring filter  $C(z)$  with Gaussian noise of zero mean with unit variance.

$$C(z) = \frac{0.1}{1 - 0.9z^{-1}} \quad (18)$$

All simulation examples are done with 100 trial.  $g(n)$  represent additive Gaussian noise of zero mean with variance of  $10^{-3}$ .

The all Gaussian noise are uncorrelated with among them. Another conditions are summarized as follows.

[Proposed algorithm]

FFT size:  $M = 64$

Forgetting constant:  $\alpha = \lambda = 0.9999$

[Full tap LMS]

Tap length:  $K = 400$

Number of adaptive weights:  $N_L = 400$

Step size:  $\mu = 0.005$

[Sparse tap LMS]

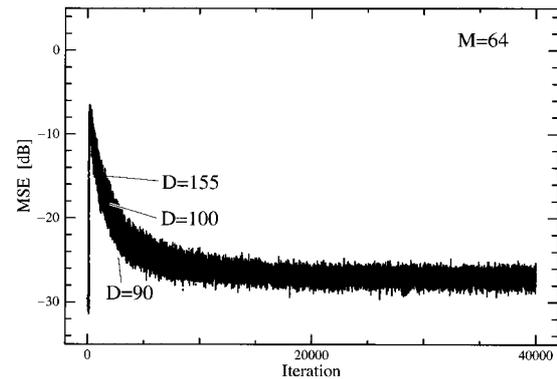
Tap length:  $K = 400$

Number of adaptive weights:  $N_L = 64$

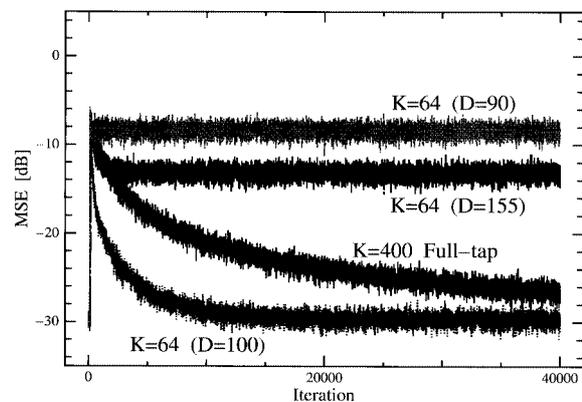
Number of Fixed delay:  $D_L = 100$

Step size:  $\mu = 0.04$

In the LMS algorithm, step size  $\mu$  is chosen as fastest and stable from the experimentation. In the full tap LMS algorithm, the adaptive filter has entire adaptive weights on their taps, whereas sparse tap algorithm only has  $N_L$  taps as shown in Fig. 2. In addition, the sparse tap LMS adaptive filter equipped with fixed



(a) Proposed ( $M = 64$ ).



(b) LMS algorithm.

Fig. 12 MSE convergence characteristics.

delay  $D_L = 100$  to concentrate the dispersive response region.

Figures 12(a) and (b) show MSE convergence of the proposed algorithm and LMS algorithms, respectively. The definition of the MSE is given by

$$\text{MSE} = 10 \log_{10} E[e^2(n)] \quad (19)$$

where  $n$  represents time index.

From Figs. 12(a) and (b), the proposed algorithm as fast as the sparse tap LMS algorithm with fixed delay  $D = D_L = 100$ . Regardless of the value of  $D$ , proposed algorithm shows robustness as shown in Fig. 12(a).

The sparse tap LMS algorithm with fixed delay converges faster than the full tap LMS algorithm. On the other hand, if the fixed delay  $D_L$  does not agree with the delay  $D$ , the residual error becomes quite large in the case of  $D = 90$  and  $D = 155$ . By applying the tap selection algorithm [5]–[7] to the sparse tap adaptive filter enables significantly improvement of the convergence property. However, the penalty of the miss adjustment in the tap selection algorithm is not always small.

## 7. Conclusion

In this paper, we proposed the use of the frequency domain adaptive algorithm based on the frequency domain sampling theorem, to estimate impulse response with a flat delay and dispersive region. The proposed technique enables efficient identification of the impulse response with a flat delay and dispersive region.

Since the proposed technique can evaluate the sub-band signal in equally spaced discrete frequency points, regardless of the flat delay with length  $D$ , it can reduce both the number of adaptive weights and the complexity of the flat delay estimation.

In addition, the flat delay estimation and the impulse response estimation of the dispersive region can be performed independently. Thus the impulse response estimation accuracy is not disturbed by the changing of the flat delay  $D$ .

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