LETTER Special Section of Selected Papers from the 12th Workshop on Circuits and Systems in Karuizawa

Design of Integer Wavelet Filters for Image Compression

Hitoshi KIYA[†], **Hiroyuki KOBAYASHI**[††], *Members*, *and* **Osamu WATANABE**[†], *Student Member*

SUMMARY This paper discusses a method of designing linear phase two-channel filter banks for integer wavelet transform. We show that the designed filter banks are easily structed as the lifting form by leading relationship between designed filters and lifting structure. The designed integer wavelets are applied to image compression to verify the efficiency of our method.

key words: *JPEG2000, integer wavelet, integer coefficients filter, lifting*

1. Introduction

This paper discusses discrete-time wavelet transform with integer filters, called integer wavelet transform, and its application to still image compression. Integer wavelet transform has been investigated since it can be carried out fast and effectively by shift-and-add operations. Moreover,the integer wavelet transform with lifting structure is suitable for both lossless and lossy image compressions [1]–[3].

This paper has two purposes. The first is to present a method of designing two-channel linear phase filter banks composed of filters with integer coefficients, since it is well known that wavelet transform can be carried out based on filter banks [4]. The method is based on the method in Refs. [5], [6] and [7], which was developed for filter banks with floating coefficients. Thus, this paper shows that the method can also be used for filter banks with integer coefficients. Although conventional methods of designing filter banks with integer coefficients are restricted to factorization of maximally-flat filters into [8], our method does not have this restriction. Thus, we can obtain different filter banks which cannot be obtained by conventional methods.

The second purpose is to verify that the filter banks obtained by this method are effective for image compression using integer wavelet transform. A couple of simulations under the JPEG 2000. standard with lifting structures[2] are discussed.

Manuscript received June 28, 1999.

[†]The authors are with the Department of Electrical, Engineering, Tokyo Metropolitan University, Hachioji-shi, 192-0397 Japan.

[††]The author is with the Department of Electrical, Engineering, Tokyo Metropolitan College of Technology, Tokyo, 140-0011 Japan.

2. Design Procedure

Figure 1 shows a block diagram of a two-channel filter bank. One of our purposes is to show how to design linear phase filters $H_0(z)$ and $H_1(z)$ that have integer coefficients and satisfy the perfect reconstruction conditions,

$$H_0(z)H_1(-z) - H_1(z)H_0(-z) = 2z^{-L}, \quad (1)$$

$$F_0(z) = H_1(-z), \quad (2)$$

$$F_1(z) = -H_0(-z). \quad (3)$$

We use the design method shown in Refs. [5]–[7], which was proposed for filter banks with floating coefficients.

We first design an analysis highpass filter $H_1(z)$, and then design an analysis lowpass filter $H_0(z)$ using $H_1(z)$. We assume that passband amplitude of the analyzer lowpass filter $H_0(z)$ is 1, and that of the analyzer highpass filter $H_1(z)$ is 2, since the filters are suitable for lifting structure.

2.1 Integer Filters

Let us define integer filters. Integer filters enable us to carry out all calculations by shift-and-add operations. A Nth-order integer filter $H(z)$ is described as follows.

$$H(z) = (a_0 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_N z^{-N})/2^k, \quad (4)$$

where a_i, $(i = 0, 1, \cdots, N)$, N and k are integer numbers.

2.2 How to Design Filters

Our design method is summarized as follows.

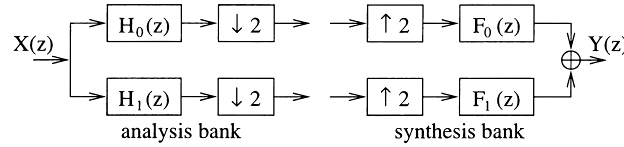


Fig. 1 Two-channel filter bank.

Step 1: Designing a half-band highpass filter $H_1(z)$.

1-1 Design an order D linear phase lowpass filter $H_1'(z)$ whose passband amplitude is equal to 1.
1-2 Modify $H_1'(z)$ in the form

$$H_1(z) = z^{-D} - H_1'(z^2). \quad (5)$$

The filter $H_1(z)$ becomes a $2D$-th order type-I (deg $H_1(z)$ =odd, $H_1(z)$ has a symmetrical impulse response) highpass filter whose passband amplitude is 2. It is easily shown that $H_1(z)$ in Eq. (5) has integer coefficients if $H_1'(z)$ is an integer filter.

For example, when we choose $H_1'(z) = (-1 + 9z^{-1} + 9z^{-2} - z^{-3})/2^4$, the highpass filter $H_1(z)$ is given by

$$H_1(z) = (1 - 9z^{-2} + 16z^{-3} - 9z^{-4} + z^{-6})/2^4. \quad (6)$$

Its passband amplitude, $|H_1(\pi)|$, is equal to 2.

Step 2: Choosing an initial solution $H_0'(z)$.

Next, we choose an initial solution $H_0'(z)$ which satisfies Eq. (1) with $H_1(z)$. In this paper, we choose

$$H_0'(z) = 1 \quad (7)$$

as one of the solutions. $H_0'(z)$ and $H_1(z)$ always satisfy Eq. (1) since $H_1(z)$ is a halfband filter.

Step 3: Designing a lowpass filter $H_0(z)$

We determine a lowpass filter $H_0(z)$ in the form

$$H_0(z) = H_0'(z)z^{-J} + f(z)H_1(z). \quad (8)$$

When $f(z)$ and J satisfy the conditions below, $H_0(z)$ and $H_1(z)$ give a linear-phase perfect reconstruction filter bank.

A: $f(z)$ is a $2K$th order (K is an arbitrary odd number) type-I linear phase filter and satisfies

$$f(z) = f(-z). \quad (9)$$

B: $J = (2D + 2K)/2$.

It can also be shown that $H_0(z)$ in Eq. (8) has integer coefficients, if $H_0'(z)$ in Step 2 and $f(z)$ in Step 3 are integer filters.

When we choose $f(z) = (-1 + 5z^{-2} + 5z^{-4} - z^{-6})/2^4$, we obtain $H_0(z)$ since Eqs. (6), (7) and (8),

$$\begin{aligned} H_0(z) = (&-1 + 14z^{-2} - 16z^{-3} - 31z^{-4} + 80^{-5} \\ &+164z^{-6} + 80z^{-7} - 31z^{-8} \\ &-16z^{-9} + 14z^{-10} - z^{-12})/2^8. \end{aligned} \quad (10)$$

Its passband amplitude, $|H_0(0)|$, is equal to 1.

The filter pairs in Eqs. (6) and (10) are prototype filters investigated in [2] (referred to as 'CRF(13,7)' in the reference).

Some of integer wavelet filters which was previously reported can be lead by the proposed method. In addition to this, it enables us to obtain new integer wavelet filters.

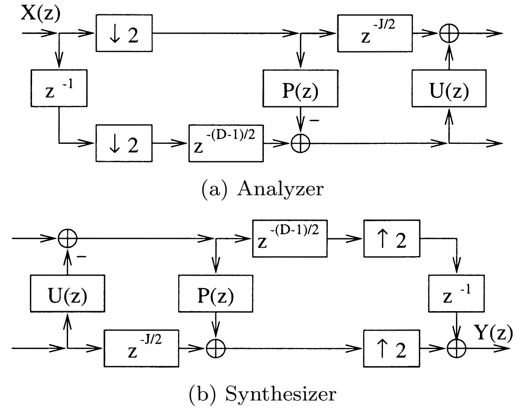


Fig. 2 Lifting structure of two-channel filter bank.

3. Relationship between Designed Filters and Lifting Structure

Figure 2 shows a lifting structure of two-channel filter banks [3]. This structure enables us to realize the filter bank effectively and to guarantee perfect reconstruction of the filter bank. In JPEG 2000, integer wavelet will be realized by lifting structure.

From the relationship between Fig. 1 and Fig. 2, we obtain the following equations.

$$H_1(z) = -P(z^2) + z^{-D}, \quad (11)$$

$$\begin{aligned} H_0(z) &= \{z^{-J} - P(z^2)U(z^2)\} + z^{-D}U(z^2) \\ &= z^{-J} + U(z^2)\{-P(z^2) + z^{-D}\} \\ &= z^{-J} + U(z^2)H_1(z). \end{aligned} \quad (12)$$

From these equations, Eqs. (5) and (8), we can show

$$P(z) = H_1'(z), \quad (13)$$

$$U(z) = f(z^{\frac{1}{2}}), \quad (14)$$

provided that $H_0'(z) = 1$. These equations mean that the designed filter banks are easily structured as the lifting form. As a result, we can consider effective transfer functions in the lifting structure when we design filter banks.

4. Design Examples and Simulation

4.1 Design Examples

In this subsection, we show some filters designed by our

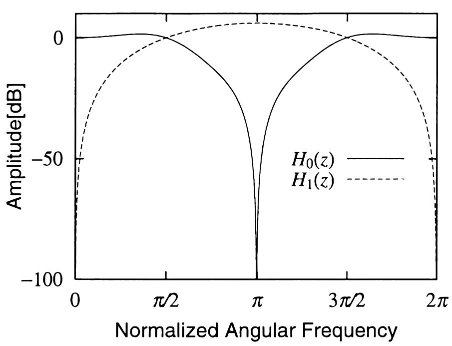
Table 1 Impulse responses of designed filters.

(a) Example 1 (order = 12, 2)

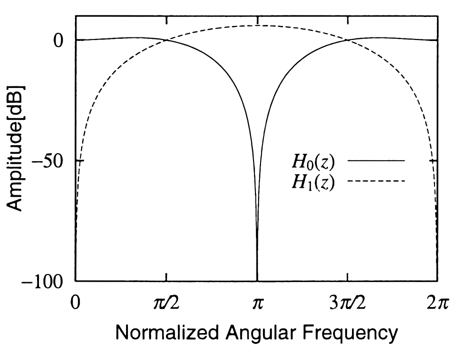
$H_1'(z)$	$[1,1]/2^1$
$H_1(z)$	$[-1,2,-1]/2^1$
$f(z)$	$[1,-5,36,36,-5,1]/2^7$
$H_0(z)$	$[-1,2,4,-10,-31,72,184,72,-31,-10,4,2,-1]/2^8$

(b) Example 2 (order = 8, 2)

$H_1'(z)$	$[1,1]/2^1$
$H_1(z)$	$[-1,2,-1]/2^1$
$f(z)$	$[1,63,63,1]/2^8$
$H_0(z)$	$[-1,2,-64,126,386,126,-64,2,-1]/2^9$



(a) Example 1



(b) Example 2

Fig. 3 Frequency responses of the designed filters.

method. Table 1 shows the filter coefficients as vector forms. Examples 1 and 2 in the table are filters designed by our method. Their frequency responses are shown in Fig. 3.

4.2 Applying the Designed Filters to Image Compression

We applied the designed filters and the sets of filters , MIT(9,7), (9,3), CDF(5,3), and CRF(13,7), to still image compression under the JPEG2000 standards with lifting structures [2]. MIT(9,7), (9,3), CDF(5,3), and CRF(13,7) are filters investigated in [2].

The gray-scale images we used were ‘Bike,’ ‘Cafe,’ ‘Woman’ (2048 × 2560 pixels) and ‘Target’ (512 × 512 pixels).

First, we calculated Peak Signal Noise Ratio (PSNR) between the original image and the lossy compressed images using implicit scalar quantization(Table 2). In these tables, each column means the PSNR using the filters.

Next, we verified the case of lossless image compression. Table 3 shows the bit rates of lossless compressed images.

Finally, we evaluated the calculation complexity for executing the wavelet transform. Table 4 shows the number of calculations of the filter banks.

From these tables we obtain the following results.

(a)Example 1(13,3)

In the lossy compression, the filter of example 1 has the almost same performance as the (9,3) filter and the best performance after the CRF(13,7) filter in the PSNR evaluation. Moreover we can see that the CRF(13,7) filter needs 2 multipliers to execute wavelet transform, although this designed filter does not need any multiplier . This is shown at Table 4. When the bit rate was high, this filter also performed best for all images except the ‘target’ image.

In the lossless compression,the bit rates of this filter are lower than the (9,3) filter for all images except the ‘target’ image.

(b) Example 2(9,3)

In the lossy compression,the filter of example 2 has almost the same performance as the CDF(5,3) filter.

In the lossless compression, the bit rates of this filter are always lower than those of the CDF(5,3) filter. This filter has the best performance for some images.

The CDF(5,3) filter has the lowest computational complexity, while the filter of example 2 has the next lowest complexity,although the difference is very small.

5. Conclusion

In this work, we design some integer filters for discrete wavelet transforms. The filters are not restricted to factors of maximally-flat filters and designed filter banks are easily constructed by the lifting structure. As a result, It becomes possible to obtain some filters with better performance than those of the conventional filters.

Acknowledgement

The authors wish to thank Mr. Takahiro Furukawa and Seiji Kimura of Sony Corporation for their useful advice.

This work was supported in part by Grant-in-Aid for Scientific Research No. 11875088 from the Japan Society for the Promotion of Science.

Table 2 PSNR between the original image and the compressed images [dB].

(A) Bike

Rate	Example 1 (13,3) (a)	Example 2 (9,3) (b)	MIT(9,7) (c)	(9,3) (d)	CDF(5,3) (e)	CRF(13,7) (f)
0.0625	23.54	23.38	23.46	23.57	23.39	23.73
0.125	26.16	26.01	26.09	26.18	26	26.38
0.25	29.35	29.25	29.24	29.37	29.21	29.55
0.5	33.17	33.1	33.06	33.17	33.11	33.31
1	37.58	37.49	37.45	37.53	37.49	37.66
2	42.96	42.89	42.74	42.94	42.96	42.86

(B) Cafe

Rate	Example 1 (13,3) (a)	Example 2 (9,3) (b)	MIT(9,7) (c)	(9,3) (d)	CDF(5,3) (e)	CRF(13,7) (f)
0.0625	18.95	18.82	18.8	18.95	18.83	18.98
0.125	20.56	20.43	20.46	20.57	20.44	20.67
0.25	22.96	22.86	22.86	22.96	22.88	23.08
0.5	26.56	26.46	26.5	26.56	26.48	26.7
1	31.82	31.78	31.8	31.8	31.78	31.96
2	38.65	38.64	38.54	38.63	38.65	38.63

(C) Woman

Rate	Example 1 (13,3) (a)	Example 2 (9,3) (b)	MIT(9,7) (c)	(9,3) (d)	CDF(5,3) (e)	CRF(13,7) (f)
0.0625	25.3	25.12	25.16	25.32	25.1	25.47
0.125	27.1	26.93	26.99	27.11	26.9	27.3
0.25	29.59	29.48	29.65	29.61	29.41	29.93
0.5	33.19	33.11	33.24	33.19	33.03	33.47
1	37.84	37.8	37.81	37.83	37.73	38.02
2	42.95	42.88	42.82	42.94	42.95	42.9

(D) Target

Rate	Example 1 (13,3) (a)	Example 2 (9,3) (b)	MIT(9,7) (c)	(9,3) (d)	CDF(5,3) (e)	CRF(13,7) (f)
0.0625	19.92	19.25	19.71	20.12	19.3	20.48
0.125	22.93	22.38	22.33	23.36	22.42	24.22
0.25	27.25	26.75	27.13	27.67	26.82	28.83
0.5	33.54	33.12	33.79	33.99	33.13	35.02
1	43.04	42.87	42.79	43.18	42.82	43.44
2	58.26	60.57	57.6	58.81	60.88	58.11

Table 3 Bitrates of lossless compressed images [bit/pixel].

Image	Example 1 (13,3) (a)	Example 2 (9,3) (b)	MIT(9,7) (c)	(9,3) (d)	CDF(5,3) (e)	CRF(13,7) (f)
bike	4.44895	4.43844	4.42338	4.45411	4.44044	4.41897
cafe	5.2466	5.23033	5.21354	5.25316	5.23189	5.2225
target	2.20563	2.14722	2.26202	2.17719	2.14795	2.24756
woman	4.4249	4.41398	4.38445	4.43015	4.41584	4.38553

Table 4 Complexities of filter banks (The number of calculations per a sample).

filter	Adds	Shifts	Multiplies
Example 1(13,3)	10	5	0
Example 2(9,3)	7	3	0
MIT(9,7)	8	2	1
(9,3)	8	4	0
CDF(5,3)	6	2	0
CRF(13,7)	10	2	2

References

[1] A.R. Calderbank, I. Daubechies, W. Sweldens, and B.L. Yeo, "Lossless image compression using integer to integer wavelet transform," IEEE International Conference on Image Processing, vol.1, no.385, pp.596–599, Oct. 1997.

[2] ISO/IEC JTC 1/SC 29/WG 1 N1143: Coding of Still Pictures JPEG 2000 Verification Model Version 3.2A, Feb. 1999.

[3] I. Daubechies and W. Sweldens, "Factoring wavelet transforms into lifting steps," J. Fourier Anal. Appl., vol.4, pp.247–269, 1998.

[4] P.P. Vaidyanathan, Multirate Systems and Filter Banks, Prentice Hall, Englewood Cliffs, New Jersey, 1993.

[5] H. Kiya, M. Yae, and M. Iwahashi, "A linear phase two-channel filter bank allowing perfect reconstruction," IEEE Proc. International Symp. on Circuits and Systems, pp.951–954, May 1992.

[6] H. Kiya, M. Yae, and M. Iwahashi, "A linear phase two-channel filter bank allowing perfect reconstruction," IEICE Trans. Fundamentals, vol.E76-A, no.4, pp.243–245, April 1993.

[7] K. Kurosawa, K. Yamamoto, and I. Yamada, "A simple design method of perfect reconstruction QMF banks," IEEE Trans. on Circuits and Systems–II, Analog and Digital Signal Processing, vol.41, no.3, March 1994.

[8] M. Ikehara, T.D. Tran, and T.Q. Nguyen, "Generalized lapped biorthogonal transforms with integer coefficients," The 13th Digital Signal Processing Symposium of IEICE, B3–2, pp.207–212, Nov. 1998.

[9] H. Kiya, H. Kobayashi, and O. Watanabe, "Design of integer wavelet filters for image compression," The 12th Workshop on Circuits and Systems in Karuizawa of IEICE, pp.73–78, April 1999.