A New Structure of Lifting Wavelet for Reducing Rounding Error

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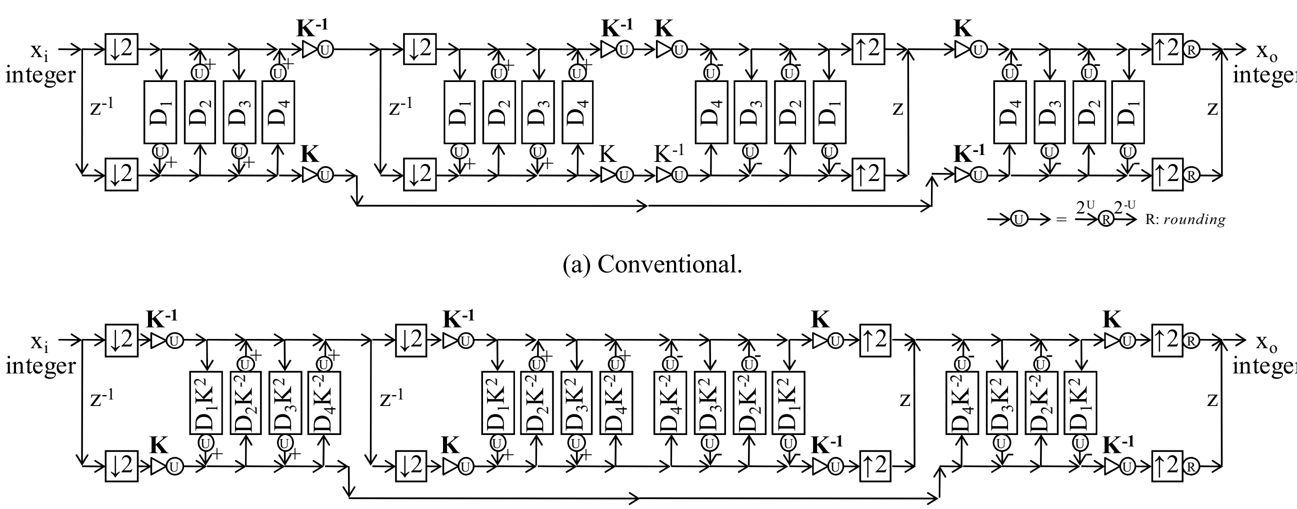
***Abstract*— The 5/3 wavelet transform with double lifting steps in JPEG 2000 can reconstruct a signal without any loss. It has been utilized for lossless coding. The 9/7 wavelet transform contains two more lifting steps and scaling operations to improve performance for lossy coding. The loss is due to 1) quantization of band signals, 2) rounding of signals after scaling and 3) finite word length expression of scaling coefficients. This paper analyzes conditions on word length of coefficients and bit depth of rounded signals for no loss. It also proposes a new structure of lifting wavelet by changing order of the lifting step and the scaling. As a result, the rounding error is not scattered by the lifting steps and the error is minimized in mini-max sense.**

I. INTRODUCTION

Over the past few years, a considerable number of studies have been made on the lifting wavelet transforms [1,2]. The 5/3 wavelet in JPEG 2000 (JP2K) [3] e.g. can reconstruct signals without any loss, when quantization is not applied. However freedom of parameters of this type is limited since it has only two lifting steps. When the lifting steps are added to increase parameters, it is necessary to introduce scaling operations to control the gain. The 9/7 wavelet in JP2K is an example of this type and it has been utilized for efficient lossy coding. The loss is due to 1) quantization of band signals, 2) rounding of signals into finite bit depth (BD) after the scaling [4,5] and 3) truncation of scaling coefficients into finite word length (WL) [6,7]. These are necessary for digital computation and entropy coding. Since input and output are expressed in integers, it is possible to get rid of the loss by assigning enough BD to the signals inside the wavelet. However it increases hardware cost and high demand to maintain compatibility between an encoder and a decoder. It is important to discuss lossless condition of the transform when numerical precision is necessary to be guaranteed. In case of discrete cosine transform (DCT), compatibility is discussed and defined by [8,9].

In this paper, we analyze 1) a condition on WL of scaling coefficients and 2) a condition on BD of rounded signals inside the wavelet transform to guarantee losslessness. It becomes possible to guarantee perfect compatibility between forward transform and backward at the minimum cost of WL and BD for any signals. Next, we propose a new structure of the lifting wavelet by bartering lifting steps and scaling $\{K^{-1}, K\}$ as illustrated in figure 1. It is also extended to variations in figure 2. As a result, the minimum BD and WL for the losslessness are decreased since the lifting steps between analysis part and synthesis part are canceled. We also investigate lossless property for specific signals such as DC signals (DC lossless) [10] of the proposed wavelet transform.



(a) Conventional.

(b) Proposed (type 1A).

Figure 1 Lifting wavelet transforms. Lifting step L and scaling K are bartered to reduce rounding errors.

II. Analysis of Lossless Conditions

In this section, we derive 1) a condition on word length (WL) of scaling coefficients and 2) a condition on bit depth (BD) of signals in the lifting wavelet transform. In case of both of them are satisfied, there is no difference between input signal to the encoder and output signal from the decoder and compatibility is guaranteed for any signals.

A. Word Length Condition (WL-C)on a Scaling

As illustrated in figure 1, the wavelet transform contains scaling with coefficients K and K^{-1}. Denoting a scaling coefficient by h, its value is rounded into h^* with finite word length WL [bit] by,

$$h^* = h + \Delta h = R[h2^{WL}]2^{-WL}. \tag{1}$$

In addition, signals to be scaled are also rounded into finite bit depth (BD). Figure 3(a) illustrates a scaling by h of an integer input value x_i. Scaling result is rounded into an integer x_o. This is a mapping of x_i to x_o defined by

$$f_h : x_i \mapsto x_o = f_h(x_i) = R[hx_i], \quad x_i, x_o \in \mathbf{Z}. \tag{2}$$

In this paper, the WL condition (WL-C) is defined by

$$f_h(x_i) - f_{h^*}(x_i) = 0, \tag{3}$$

$$\forall x_i \in \{1,2,3,\cdots,2^{BIi}\} \in \mathbf{N}.$$

As far as this condition is satisfied, the mapping is invariant to the finite word length expression of the scaling coefficient h.

This is expanded to scaling of a quotient x_i with BI_i bit integer and BF_i bit fraction as illustrated in figure 3(c). In this case, the WL-C becomes

$$f_h(w_i 2^{dF}) - f_{h^*}(w_i 2^{dF}) = 0, \tag{4}$$

$$\forall w_i = x_i 2^{BIi} \in \{1,2,3,\cdots,2^{BIi+BFi}\} \in \mathbf{N}, \quad x_i \in \mathbf{Q}.$$

This is equivalent to

$$|\Delta h| < (\Delta[hw_i 2^{dF}] + 2^{-1})/(w_i 2^{dF}) \tag{5}$$

where

$$\Delta[hw_i 2^{dF}] = \left| R[hw_i 2^{dF}] - hw_i 2^{dF} \right|.$$

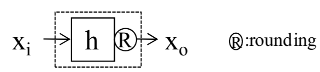
In case of $\Delta[hw_i 2^{dF}] = 2^{-1}$, it implies

$$|\Delta h| < 2^{-(BIi+BFi+dF)} \tag{6}$$

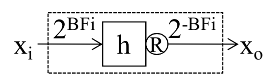
and therefore, from equation (1),the minimum WL is given by

$$WL > BI_i + BF_i + dF. \tag{7}$$

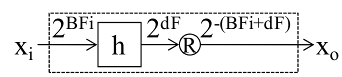
Simulation examples are summarized in figure 4. In short, the longer the input BD (=BI+BF), the longer the minimum WL.



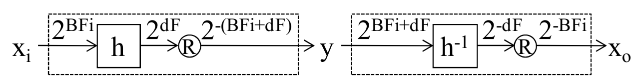
(a) Scaling of an integer.



(b) Scaling of a quotient with $\mathbf{BF_i}$ fractional bit.

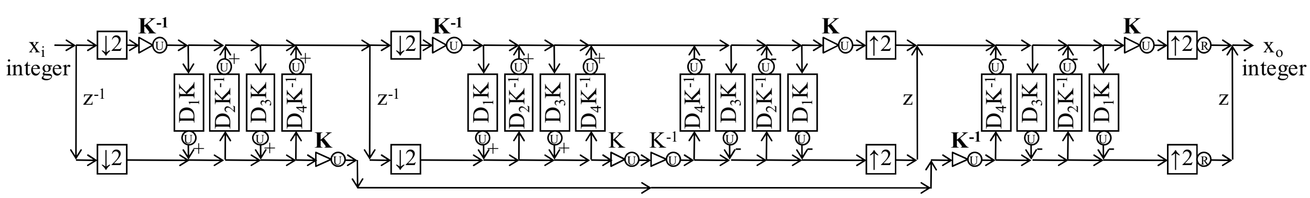


(c) Increment of fractional digits by **dF** bit.

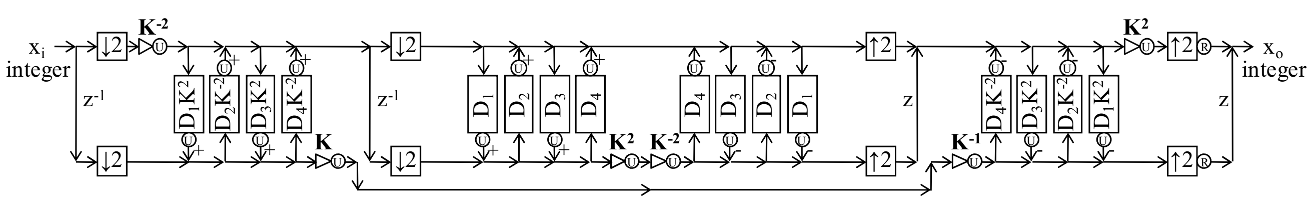


(d) A scaling pair for a quotient with **dF** bit increment.

Figure 3 Scaling operations in the system.



(a) Proposed (type 2A).



(b) Proposed (type 2B).

Figure 2 Variations of the proposed wavelet transform.

B. Bit Depth Condition (BD-C) on a Scaling Pair

In case of the lifting wavelet in figure 1, scaling is always performed in a pair, e.g. K^{-1} in forward transform and K in backward. This scaling pair is illustrated in figure 3(d). Defining the error by

$$\Delta x = x_o - x_i \ , \tag{8}$$

its probability density function has a unique property:

$$\begin{cases} p(\Delta x = e) \neq 0, & e \in \{0, \pm 2^{-BFi}\} \\ p(\Delta x = e) = 0, & e \notin \{0, \pm 2^{-BFi}\} \end{cases}, \tag{9}$$

$$\max|\Delta x| = 2^{-BFi} \, . \tag{10}$$

On the other hand, the BD condition (BD-C) is satisfied if and only if the composite mapping g_h defined by

$$g_h = f_{1/h} \circ f_h : x_i \mapsto y \mapsto x_o \tag{11}$$

is bijective. As a result, for the scaling pair, it is equivalent to "f_h is injective" and also "$f_{1/h}$ is surjective". It is satisfied when

$$dF > -\log_2 h \ . \tag{12}$$

For example, when

$$\begin{cases} h = K^{-1} = 0.8129 & \Rightarrow \quad dF = \lceil +0.2989 \rceil = 1 \\ h = K = 1.2302 & \Rightarrow \quad dF = \lceil -0.2989 \rceil = 0 \end{cases} \tag{13}$$

is satisfied, the scaling pair becomes lossless.

The proposed lifting wavelet in figure 1(b) utilizes the properties in equation (10) and (13) under the WL-C in equation (4).

III. Proposed Lifting Wavelet

In this section, we confirm that the proposed wavelet structure can reduce WL cost and BD cost under both of the WL-C and BD-C for AR(1) signals. We also investigate losslessness for specific signals (DC lossless property) of the proposed wavelet transform.

A. Near Losslessness

Figure 5 illustrates PSNR of the reconstruction error x_o-x_i to the entropy rate of all of the band signals. The fraction bit U is varied from a negative integer to a positive integer. The WL-C is satisfied. Input signal is the AR(1) model with 8 [bit] integer values (BI_x=7, BS_x=1). It is confirmed that the type 2B can be used for lossless coding. Figure 6 illustrates the maximum absolute value of the reconstruction error to the fraction bit U. Since the maximum error is limited to one for type 2B, it can be used for near lossless coding.

B. The Minimun Bit Depth (min.BD)

According to figure 6, it is confirmed that the conventional wavelet requires $U \geq 4$ [bit] to guarantee losslessness for AR(1) signals. On the other hand, the proposed wavelets require $U \geq 3$, 3 and 2 [bit] for type 1A, 2A and 2B respectively. As a result, the minimum fraction bit is reduced from 4 [bit] to 2 [bit] at maximum. Compatibility and compression ratio are illustrated in figure 7 and 8 respectively.

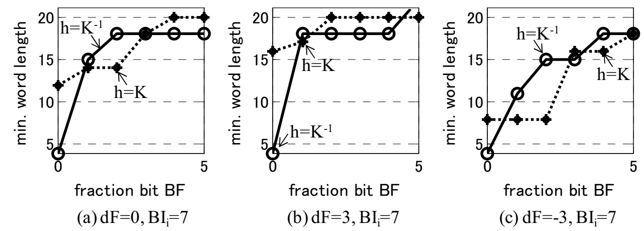


Figure 4 Minimum word length of scaling coefficients. Mapping of a scaling is invariant under this condition.

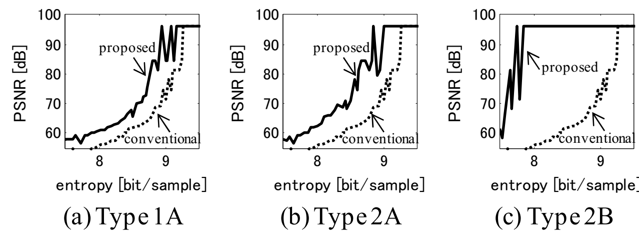


Figure 5 Rate distortion curves around near lossless range. Type 2B becomes lossless at the minimum entropy rate.

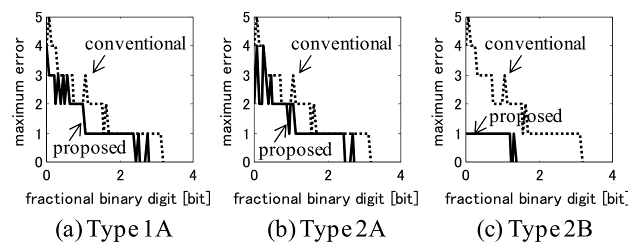


Figure 6 Maximum absolute value of the error for U.

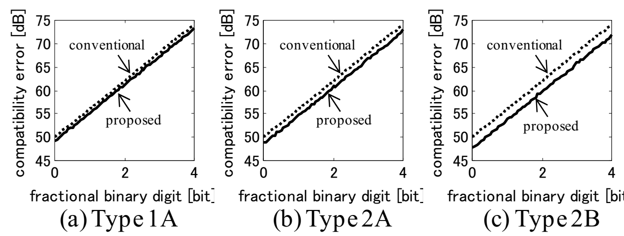


Figure 7 Compatibility defined by difference of band signals from the ideal transform with long enough bit depth.

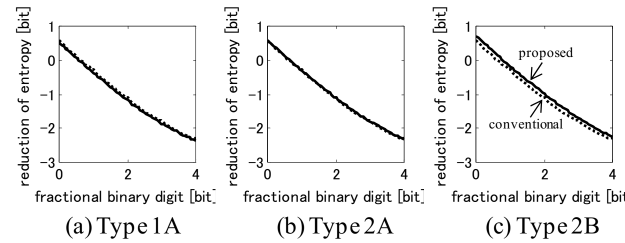


Figure 8 Reduction of entropy rate. It is proportional to fractional bit U and trade off with the compatibility.

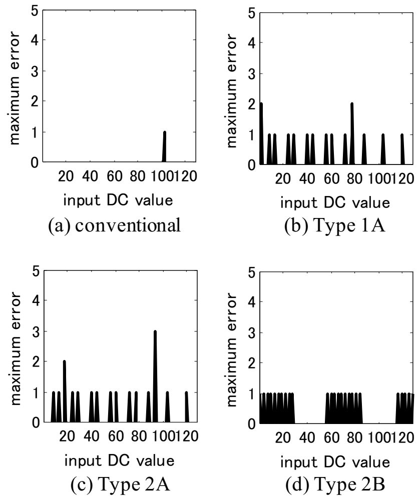


Figure 9 Maximum error values for constant (DC) signals with a value at horizontal axis.

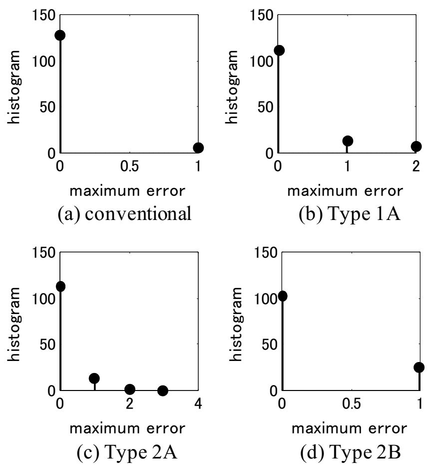


Figure 10 Histogram of the reconstruction error for DC input.

C. The Minimum Word Length (min.WL)

Figure 4 and figure 6 give the minimum WL of a scaling coefficient. For K^{-1} in the 1st stage of the forward transform e.g., (BI_i, BF_i dF)=(BI_x, U, 0) for the conventional and (BI_x, 0, U) for the proposed type 1A. Since (BI_i, BF_i, dF)=(7, 4, 0) and (7, 0, 3) for the conventional and the proposed, the minimum WL is given by 18 [bit] and 4 [bit] respectively according to figure 4. It is reduced by 14 [bit].

D. Specific Losslessness

Even though the BD-C is not satisfied, U=0 in figure 1 and 2 for example, there is no error on specific input signals. Figure 9 indicates the reconstruction error for a constant value (DC) input signals. This is a while balance for evaluation of compatibility between an encoder and a decoder. Maximum value of the reconstruction error is indicated to each of the input DC values. The conventional wavelet transform becomes lossless for 99.22 [%] of input values. In case of the proposed wavelets, the ratios are 88.28 [%], 88.28 [%], 80.47 [%] for type 1A, type 2A, type 2B, respectively.

Figure 11 summarizes histogram of the errors. It is confirmed that the proposed type 2B is the best in mini-max sense. In this type, error occurs in the scaling pair of (K^{-2}, K^{2}) at the 1st stage. Equation (10) implies the maximum value of the error is one as BF_i=U=0. For DC inputs, error does not occur in high pass band since signal value in this band is zero. The set of lifting steps in forward and backward transform is equivalent to the transfer function of unity. As a result, the error is preserved at the final output of the backward transform. On the other hand, the error is scattered by the lifting steps in other transforms.

IV. Conclusions

In this paper, we have derived the WL-C and the BD-C for guaranteeing losslessness for any input signals. We have proposed a new lifting wavelet transform and its variations in cascade form by changing order of the lifting step and the scaling. It was confirmed that the proposed type 2B wavelet transform decreases the minimum bit depth (BD) from 4 [bit] to 2 [bit] under the WL-C and the BD-C. It was also confirmed that the minimum word length (WL) of a multiplier was reduce from 18 [bit] to 4 [bit] by the proposed wavelet.

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