MULTIPLE-PEAK MODEL FITTING FUNCTION FOR DCT SIGN PHASE CORRELATION WITH NON-INTEGER SHIFT PRECISION

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ABSTRACT

We propose two fitting functions for shift estimation using discrete cosine transform sign phase correlation (DCT-SPC) with non-integer accuracy. The DCT-SPC can be used in order to estimate the shift values and the similarity between two signals. However, the estimated shift values are limited to integer numbers. The proposed fitting functions enable the DCT-SPC to estimate the non-integer shift values using only the sign of the DCT coefficients. The multiple-peak model in the proposed fitting functions provides more accurate values than other models. Simulations are presented to demonstrate the effectiveness of the proposed fitting functions.

Index Terms— phase-only correlation, DCT sign phase correlation, registration, non-integer accuracy, fitting function

1. INTRODUCTION

The DCT sign phase correlation (DCT-SPC) that we proposed previously is used to estimate the shift values and the similarity between two signals [1][2]. DCT-SPC uses only the signs of discrete cosine transform (DCT) coefficients of signals. Image retrieval in JPEG-coded images [3] and sign phase scrambling [4] are the applications of DCT-SPC, and the effectiveness of DCT-SPC is confirmed. However, the shift values estimated by DCT-SPC are limited to integer numbers. In the present study, we attempt to estimate the shift values with non-integer numbers by DCT-SPC.

DCT-SPC is closely related to phase-only correlation (POC). POC or phase correlation is used to estimate the shift values and the similarity between signals [6]–[10]. The continuous expression of POC was first introduced as PHAT [5], and the POC in terms of FFT was proposed by Kuglin and Hines in 1975 [6]. Phase-only correlation uses the phase of discrete Fourier transform (DFT) coefficients and estimates the shift values based on the Fourier shift property. This concept is extended to the estimation of rotated and scaled values between images by log-polar transform. Some methods, such as interpolation [6], curve fitting [7][8], and direct calculation of transformed domain [9][10], enable POC to estimate shift values with non-integer numbers.

In the present paper, we consider curve fitting and propose two fitting functions, namely, the single-peak model and the multiple-peak model. These two fitting functions enable DCT-SPC to estimate shift values with non-integer numbers. In contrast to the single-peak model, the multiple-peak model changes its shape flexibly. As a result, the multiple-peak model matches the complex shape of the DCT-SPC function, and the estimation values obtained by the multiple-peak model are more accurate than those obtained by the single-peak model. Experimental results are presented to confirm the appropriateness and effectiveness of the proposed fitting functions.

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2. PRELIMINARY

$\mathbb{Z}$ and $\mathbb{R}$ denote the set of integer numbers and the set of real numbers, respectively.

2.1. Phase-only correlation

Let $G_i(k)$ be N-point DFT coefficients of N-point signal $g_i(n)$, $(i, k, N, n \in \mathbb{Z})$. The phase term, $\phi_{G_i}(k)$, is defined in terms of $G_i(k)$ and the absolute value, $|G_i(k)|$, as

$$\phi_{G_i}(k) = G_i(k)/|G_i(k)|. \tag{1}$$

If $|G_i(k)| = 0$, then $\phi_{G_i}(k)$ is replaced by zero. The normalized cross spectrum, $R_\phi(k)$, is given as

$$R_\phi(k) = \phi^*_{G_1}(k) \cdot \phi_{G_2}(k) \tag{2}$$

where $\phi^*_{G_1}(k)$ denotes the complex conjugate of $\phi_{G_1}(k)$.

The phase-only correlation, $r_\phi(n)$, between $g_1(n)$ and $g_2(n)$, is defined as the inverse DFT of $R_\phi(k)$:

$$r_\phi(n) = \frac{1}{N}\sum_{k=0}^{N-1} R_\phi(k) W_N^{-nk} \tag{3}$$

where W_N denotes $\exp(-j2\pi/N)$ [6].

2.2. DCT sign phase correlation

Let $G_{i_C}(k)$ be the N-point DCT coefficients of N-point signal $g_i(n)$. The DCT sign is defined in terms of $G_{i_C}(k)$ and the absolute value, $|G_{i_C}(k)|$, as

$$\sigma_{G_i}(k) = G_{i_C}(k)/|G_{i_C}(k)|. \tag{4}$$

If $|G_{i_C}(k)|$ is zero, then $\sigma_{G_i}(k)$ is replaced by zero. The DCT sign product, $R_\sigma(k)$, between $g_1(n)$ and $g_2(n)$ is given as

$$R_\sigma(k) = \sigma_{G_1}(k) \cdot \sigma_{G_2}(k). \tag{5}$$

The DCT sign phase correlation, $r_\sigma(n)$, is defined by $R_\sigma(k)$ as

$$r_\sigma(n) = \sqrt{\frac{2}{N}} \sum_{k=0}^{N-1} K_k R_\sigma(k) \cos\left(\frac{\pi n k}{N}\right) \tag{6}$$

where

$$K_k = \begin{cases} 1/2, & k = 0 \\ 1 & k \neq 0 \end{cases}. \tag{7}$$

The shift value with integer numbers is expressed by the location n of the maximum value of Eq. (6) [1][2].

2.3. Shift estimation with non-integer numbers

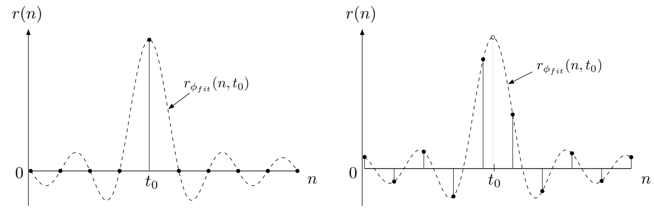
Curve fitting is a typical method of non-integer shift estimation for POC. The curve fitting requires a fitting function, which connects the continuous expression of POC with the discrete expression of POC. An example of the fitting function for POC is given as

$$r_{\phi_{fit}}(n, t_0) = \frac{\alpha}{N} \frac{\sin\big(\pi(n - t_0)\big)}{\sin\big(\pi(n - t_0)/N\big)} \tag{8}$$

where t_0 denotes the shift value ($t_0 \in \mathbb{R}$), and α denotes a constant ($\alpha \leq 1$) [8]. In the case of integer shift values, the peak location of the discrete expression of POC and that of the continuous expression of POC are the same as depicted in Fig. 1 (a), in which $r_{\phi_{fit}}(n, t_0)$ is expressed by dotted line and $r_\phi(n)$ is expressed by black points. In the case of non-integer shift values, the peak location of discrete expression and that of continuous expression are different as depicted Fig. 1 (b). Therefore, by fitting $r_{\phi_{fit}}(n, t_0)$ in Eq. (8) to $r_\phi(n)$ in Eq. (3), the shift value with non-integer numbers is estimated. As shown in Fig. 1, the fitting function in Eq. (8) is a single-peak model because of the shape. The shape of the single-peak model is fixed regardless of t_0. The goal of the present paper is to derive fitting functions for DCT-SPC.

3. MULTIPLE-PEAK MODEL FITTING FUNCTION

We propose two fitting functions by considering the relationship between DCT-SPC and POC.



(a) integer shift values (b) non-integer shift values

Fig. 1. Curve fitting

3.1. Relationship between DCT-SPC and POC

Let the $2N$-point symmetrically extended signal $\hat{g}_i(n)$ be

$$\hat{g}_i(n) = \begin{cases} g_i(n), & n = 0, 1, \cdots, N-1 \\ g_i(2N-n-1), & n = N, N+1, \cdots, 2N-1 \end{cases}. \tag{9}$$

The relationship between $\hat{R}_\phi(k)$ and $R_\sigma(k)$ is given as

$$\hat{R}_\phi(k) = R_\sigma(k), \quad k = 0, 1, \cdots, N-1 \tag{10}$$

in [1][2]. $\hat{r}_\phi(n)$ is expressed by the inverse DFT of $\hat{R}_\phi(k)$ as

$$\begin{aligned} \hat{r}_\phi(n) &= \frac{1}{2N}\left(\sum_{k=0}^{2N-1} \hat{R}_\phi(k) W_{2N}^{-nk}\right) \\ &= \frac{1}{2N}\left(\sum_{k=0}^{N-1} \hat{R}_\phi(k) W_{2N}^{-nk} + \sum_{k=1}^{N-1} \hat{R}_\phi^*(k) W_{2N}^{nk}\right) \\ &+ \frac{1}{2N} \hat{R}_\phi^*(N) W_{2N}^{nN}. \end{aligned} \tag{11}$$

Note that, as a result of symmetric signals, $\hat{R}_\phi^*(N) = 0$. From Eq. (10), we obtain

$$\hat{r}_\phi(n) = \frac{1}{N} \sum_{k=0}^{N-1} K_k R_\sigma(k) \cos\left(\frac{\pi n k}{N}\right). \tag{12}$$

Note that $K_0 = 1/2$ and $K_k = 1$ for $k \neq 0$ from Eq. (7). From Eq. (6) and Eq. (12), the relationship between $\hat{r}_\phi(n)$ and $r_\sigma(n)$ is

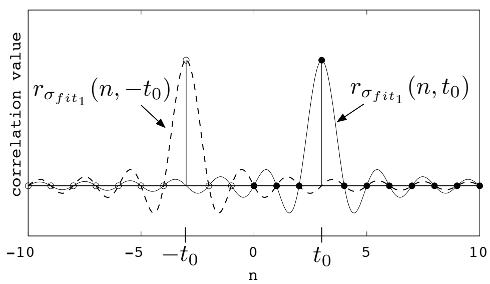
$$\hat{r}_\phi(n) = r_\sigma(n), \quad n = 0, 1, \cdots, N-1. \tag{13}$$

Therefore, the N-point DCT-SPC corresponds to the first N points of the $2N$-point POC between the symmetrically extended signals. That is, there is one peak in the first N points of the $2N$-point POC.

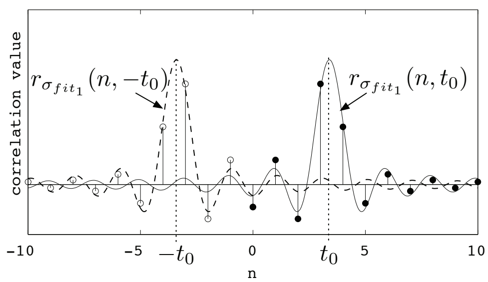
3.2. Single-peak model

From the above discussion, the single-peak model for DCT-SPC is defined by replacing N in Eq. (8) with $2N$ as follows:

$$r_{\sigma_{fit_1}}(n, t_0) = \frac{\alpha}{2N} \frac{\sin\big(\pi(n - t_0)\big)}{\sin\big(\pi(n - t_0)/2N\big)}. \tag{14}$$



(a) integer shift values



(b) non-integer shift values

Fig. 2. Interference of multiple peaks: The black and white points express $\hat{r}_\phi(n)$, and the black points correspond to $r_\sigma(n)$. Two peaks, which are expressed as one peak in DCT-SPC, affect each other in the case of non-integer shift values.

That is, the single peak model assumes that one peak appears in the $2N$-point POC.

There are two peaks in the $2N$-point POC between the symmetrically extended signals [1][2]. When the shift value is an integer, there is no effect of another peak on the value $r_\sigma(n)$, as shown in Fig. 2 (a). However, when the shift value is non-integer numbers, the two peaks affect each other, as shown in Fig. 2 (b). Hence, the single peak model is not sufficient to estimate the shift values with non-integer numbers.

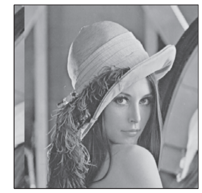
3.3. Multiple-peak model

A peak in the N-point DCT-SPC between signals is the degeneracy of two peaks in the $2N$-point POC between the symmetrically extended signals.

Considering the effect of another peak, the multiple-peak model is proposed. The multiple-peak model, $r_{\sigma_{fit_2}}(n,t_0)$, is defined using single-peak model in Eq. (14) as

$$\begin{aligned} &r_{\sigma_{fit_2}}(n,t_0) \\ &\quad= r_{\sigma_{fit_1}}(n,t_0) + r_{\sigma_{fit_1}}(n,-t_0) \\ &\quad= \frac{\alpha}{2N}\left(\frac{\sin(\pi(n-t_0))}{\sin(\pi(n-t_0)/2N)} + \frac{\sin(\pi(n+t_0))}{\sin(\pi(n+t_0)/2N)}\right). \end{aligned} \tag{15}$$

(a) image



(b) shifted image

Fig. 3. Shifted image with non-integer numbers.

The multiple-peak model has various shapes in accordance with t_0.

3.4. Fitting and symmetrical extension

The non-integer shift values between two signals are estimated by the fitting function and the DCT-SPC between signals. From the location p of the maximum value in DCT-SPC, the non-integer shift value, $\bar{t_0}$, is given by the least-squares method, i.e.,

$$\bar{t_0} = \underset{t_0}{\operatorname{argmin}} \left(\sum_{n=p-\xi}^{p+\xi} (r_{\sigma_{fit_i}}(n,t_0) - r_\sigma(n))^2 \right) \tag{16}$$

where ξ denotes the number of locations around p, and $\underset{n}{\operatorname{argmax}}(f(n))$ and $\underset{n}{\operatorname{argmin}}(f(n))$ give n, where $f(n)$ is the maximum value and the minimum value, respectively.

The value $r_\sigma(n)$ is redefined, because $r_\sigma(n)$ $(n < 0)$ is undefined in Eq. (16). In the case of the multiple-peak model, the $r_\sigma(n)$ is replaced by

$$r_\sigma(n) = \begin{cases} r_\sigma(n), & n = p-\xi \geq 0 \\ r_\sigma(-n), & n = p-\xi < 0 \end{cases}. \tag{17}$$

4. SIMULATION

We evaluated the proposed fitting functions by the non-integer shift estimation. The shifted image is generated by changing the phase of the original image (lena, 512×512, 8 bits/pixel). Fig. 3 (a) and (b) show the original image and an example of the shifted image by 30.16 in the horizontal direction, respectively. In the simulation, the shift values in the corresponding line in these images were estimated. A total of 50 lines were used, and the successive 400 points from the right end in each line were used.

Fig. 4 shows the mean of the absolute error ϵ_{t_0}, i.e.,

$$\epsilon_{t_0} = |\bar{t_0} - t_0|. \tag{18}$$

Fig. 4 shows that the two proposed fitting functions enable DCT-SPC to estimate the shift values with non-integer numbers. Moreover, the estimation values obtained by the

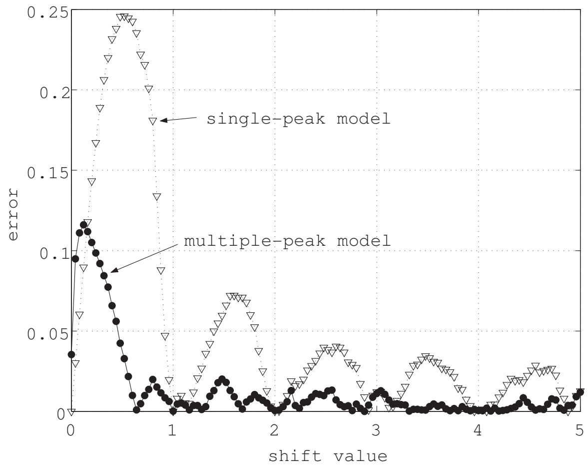
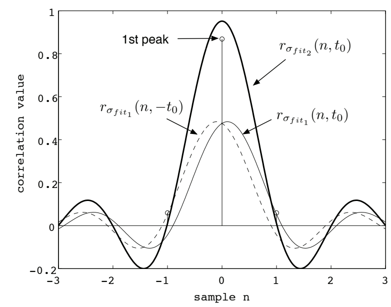


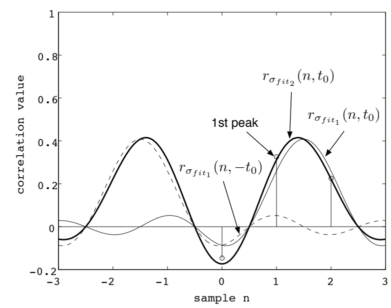
Fig. 4. Mean of absolute error, $\xi = 1$

multiple-peak model are generally more accurate than those obtained by the single-peak model.

Fig. 5 (a) and (b) show the shapes of the multiple-peak model. The shape of the multiple-peak model depends on the shift value t_0. The flexibility of the multiple-peak model provides accurate estimation.



(a) $t_0 = 0.1$



(b) $t_0 = 1.5$

Fig. 5. Shape of the multiple-peak model: Various shapes in accordance with t_0 provides accurate estimation.

5. CONCLUSION

We have proposed two fitting functions for DCT-SPC in order to estimate the non-integer shift values. Considering the relationship between DCT-SPC and POC, we have derived the fitting functions. Experimental results have been presented and the appropriateness of the proposed functions has been confirmed. The proposed functions enable DCT-SPC to estimate the shift values with non-integer numbers. The shape of the multiple-peak model is flexible. This corresponds to the complex shape of the DCT-SPC to provide more accurate values than those in the single-peak model.

6. REFERENCES

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