NON SEPARABLE 2D FACTORIZATION OF SEPARABLE 2D DWT FOR LOSSLESS IMAGE CODING

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ABSTRACT

In this report, we propose a reversible two dimensional (2D) discrete wavelet transform (DWT) compatible to the irreversible 9-7 DWT for lossy coding in the JPEG 2000. All the filters and scalings are factorized into lifting steps, and signals are rounded into integers, so that the proposed DWT becomes reversible and applicable to lossless coding. Furthermore, we factorize the separable 2D transfer function of the 2D DWT into non separable 2D functions to reduce the lifting steps and rounding operations. As a result, errors due to the rounding operations are reduced and the compatibility to the irreversible 9-7 DWT is improved.

***Index Terms*—** lossless, image, wavelet, coding

1. INTRODUCTION

The discrete wavelet transform (DWT) has been playing an important role in image and video coding since it is adopted to the JPEG 2000 international standard [1]. Recently, not only its application to high quality digital cinema [2], but also its flexibility on transcoding [3,4] between various types of pictures have been becoming technical issues of great importance.

The JPEG 2000 utilizes the reversible 5-3 DWT for lossless coding and the irreversible 9-7 DWT for lossy coding respectively [1]. Both of them are composed of the lifting structure to guarantee the perfect reconstruction property [5-7]. The former can implement the lossless coding by introducing rounding operations on signals. However its lossy coding performance is not high. The latter realizes high performance lossy coding by introducing the bit truncation or the quantization. However lossless coding is not feasible since the DWT is irreversible [8-9]. Therefore, it has becoming an important matter to construct a reversible DWT compatible to the irreversible DWT.

When these two kinds of DWTs are connected, the forward reversible 5-3 DWT and the backward irreversible 9-7 DWT for example, errors due to mismatching of frequency characteristics are observed in the reconstructed signal. To avoid this mismatching, a reversible 9-7 DWT has been proposed [10]. In this DWT (existing DWT), all the filters and the scalings are factorized into lifting steps, and signals are rounded into integers. However, when the existing DWT is connected to the irreversible 9-7 DWT, errors due to the rounding are not negligible, even though the bit truncation or the quantization is not applied.

In this report, we propose a new reversible two dimensional (2D) DWT compatible to the irreversible 9-7 2D DWT. We factorize the separable 2D transfer function of the DWT into non separable 2D functions. Basic idea of the non separable factorization had been already proposed [11-13]. However, their lifting steps had redundancy and compatibility to the 9-7 DWT was not considered. In this report, we synthesize the lifting steps maintaining the compatibility of the proposed DWT to the irreversible 9-7 DWT. As a result, the number of the rounding operations is reduced by half and the compatibility is improved.

2. SEPARABLE 2D DWT

Fig.1(a) illustrates the forward transform of the irreversible 9-7 2D DWT in the JPEG 2000 [1]. Its separable 2D transfer function is defined by

$$\mathbf{Y} = (\mathbf{J}_k \mathbf{L}_{\mathrm{H}_4^*,\mathrm{H}_3^*} \mathbf{L}_{\mathrm{H}_2^*,\mathrm{H}_1^*} \mathbf{P}_{23})(\mathbf{J}_k \mathbf{L}_{\mathrm{H}_4,\mathrm{H}_3} \mathbf{L}_{\mathrm{H}_2,\mathrm{H}_1} \mathbf{P}_{23})\mathbf{X} \quad (1)$$

for the input signals $\mathbf{X}$ and the output signals $\mathbf{Y}$;

$$\begin{matrix} \mathbf{X} = [X_{11} \quad X_{21} \quad X_{12} \quad X_{22}]^T \\ \mathbf{Y} = [LL \quad LH \quad HL \quad HH]^T \end{matrix},$$

where the filters H_n and H_n^*, $n \in \{1,2,3,4\}$, are given by

$$\begin{bmatrix} \mathrm{H}_n \\ \mathrm{H}_n^* \end{bmatrix} = \begin{bmatrix} H_n(z_1) \\ H_n(z_2) \end{bmatrix}, \begin{bmatrix} H_1(z) & H_2(z) \\ H_3(z) & H_4(z) \end{bmatrix} = \begin{bmatrix} h_1 & h_2 \\ h_3 & h_4 \end{bmatrix} \begin{bmatrix} 1+z^{-1} & 0 \\ 0 & 1+z \end{bmatrix}.$$

A set of the lifting steps $\mathrm{L}_{p.q}$ and the scaling J_k are

$$\begin{bmatrix} \mathbf{L}_{p,q} \\ \mathbf{J}_k \end{bmatrix} = \begin{bmatrix} diag[\mathbf{M}_{p,q} \quad \mathbf{M}_{p,q}] \\ diag[\mathbf{K}_k \quad \mathbf{K}_k] \end{bmatrix}, \mathbf{M}_{p,q} = \begin{bmatrix} 1 & p \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ q & 1 \end{bmatrix}$$

where a pair of scaling coefficients and other matrices are

$$\mathbf{K}_k = diag\begin{bmatrix} k^{-1} & k \end{bmatrix}, \quad \mathbf{D} = diag\begin{bmatrix} 2 & 2 \end{bmatrix},$$

$$\mathbf{P}_{23} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{P}_{24} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}.$$

Inside the irreversible DWT, signals are not rounded into integers.

Fig.1(b) illustrates the existing reversible 2D DWT [10]. The scaling $\mathbf{K}_k$ in eq.(1) is factorized into the lifting steps by

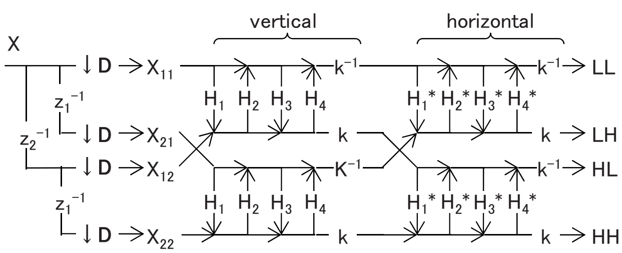
Theorem 1;

$$\mathbf{K}_k = \mathbf{M}_{s_4,s_3}\mathbf{M}_{s_2,s_1} \tag{2}$$

where

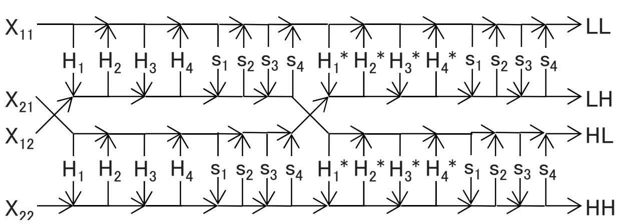
$$\begin{bmatrix} s_1 & s_2 \\ s_3 & s_4 \end{bmatrix} = \begin{bmatrix} k^{-1} & 1-k \\ -1 & 1-k^{-1} \end{bmatrix} \begin{bmatrix} ks_1 & 0 \\ 0 & (ks_1)^{-1} \end{bmatrix}$$

as illustrated in Fig.2(a). It has the same transfer function as in eq.(1). However, signals are rounded into integers just after each of the multiplications, so that it becomes reversible. Since all the filters and scalings are implemented as lifting steps, the rounding errors are canceled at output of the backward transform. However, its 32 rounding operations generate much rounding errors and decrease compatibility to the irreversible 9-7 2D DWT.



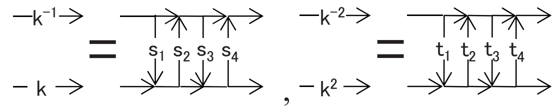
$\mathbf{D} = diag[2 \quad 2]$, $\mathrm{H}_n = H_n(z_1)$, $\mathrm{H}_n^* = H_n(z_2)$, $n \in \{1,2,3,4\}$

(a) Standard structure [1].

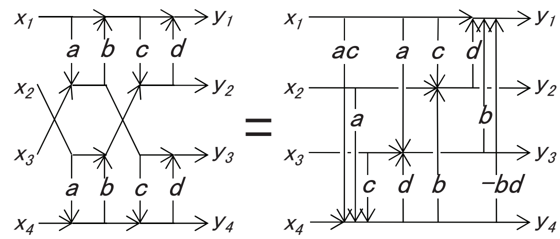


(b) Existing structure [10].

Fig.1 Separable 2D DWTs.



(a) Theorem 1



(b) Theorem 2

Fig.2 Theorems for modifications.

3. PROPOSED NON SEPARABLE 2D DWT

In this report, to reduce the number of the lifting steps and the rounding operations, we apply

Theorem 2;

$$\mathbf{L}_{d,c}\mathbf{P}_{23}\mathbf{L}_{b,a}\mathbf{P}_{23} = \mathbf{N}_{d,c,b,a} \tag{3}$$

where

$$\mathbf{N}_{d,c,b,a} = \begin{bmatrix} 1 & d & b & -bd \\ c & 1 & 0 & b \\ a & 0 & 1 & d \\ ac & a & c & 1 \end{bmatrix}$$

to eq.(1). This modification [14] unifies the eight lifting steps to the minimum four steps as illustrated in Fig.2(b). Each step has a rounding operation 'R[ ]'. In the reversible transform, the output signal is produced by

$$\hat{\mathbf{Y}} = \mathbf{X} + R\left[\mathbf{N}_{d,c,b,a}\left(\begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}\mathbf{X} + \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}\hat{\mathbf{Y}}\right)\right] \tag{4}$$

$$:= \mathbf{N}_{d,c,b,a} \circ \mathbf{X}.$$

Applying these theorems, we construct the proposed reversible 9-7 2D DWT as below.

Step 1; As illustrated in Fig.3(a), we unify the four scaling pairs $\{k^{-1}, k\}$ to only one pair $\{k^{-2}, k^2\}$ by

$$\begin{aligned}&(\mathbf{J}_k\mathbf{L}_{\mathrm{H}_4^*,\mathrm{H}_3^*}\mathbf{L}_{\mathrm{H}_2^*,\mathrm{H}_1^*}\mathbf{P}_{23})(\mathbf{J}_k\mathbf{L}_{\mathrm{H}_4,\mathrm{H}_3}\mathbf{L}_{\mathrm{H}_2,\mathrm{H}_1}\mathbf{P}_{23}) \\ &= (\mathbf{P}_{24}\mathbf{J}_k^2\mathbf{P}_{24})\mathbf{L}_{\mathrm{H}_4^*,\mathrm{H}_3^*}\mathbf{L}_{\mathrm{H}_2^*,\mathrm{H}_1^*}\mathbf{P}_{23}\mathbf{L}_{\mathrm{H}_4,\mathrm{H}_3}\mathbf{L}_{\mathrm{H}_2,\mathrm{H}_1}\mathbf{P}_{23}\end{aligned} \tag{5}$$

where

$$\mathbf{J}_k^2 = diag[\mathbf{K}_{k^2} \quad \mathbf{I}_{2\times 2}] \ .$$

Step 2; As in Fig.3(b), applying the theorem 1, we factorize the scaling into lifting steps by

$$\mathbf{K}_{k^2} = \mathbf{M}_{t_4,t_3}\mathbf{M}_{t_2,t_1} \tag{6}$$

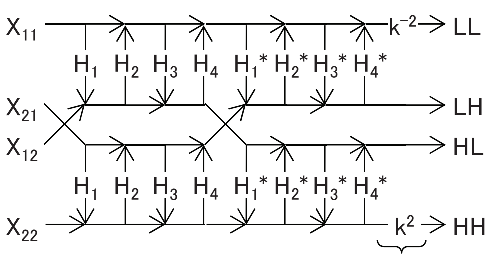
where

$$\begin{bmatrix} t_1 & t_2 \\ t_3 & t_4 \end{bmatrix} = \begin{bmatrix} k^{-2} & 1-k^2 \\ -1 & 1-k^{-2} \end{bmatrix} \begin{bmatrix} k^2 t_1 & 0 \\ 0 & (k^2 t_1)^{-1} \end{bmatrix} .$$

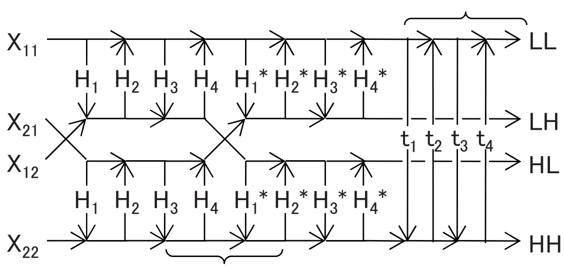
Step 3; As in Fig.3(c), applying the theorem 2, we unify the eight lifting steps to four by

$$\begin{aligned} &\mathbf{L}_{H_4^*,H_3^*}(\mathbf{L}_{H_2^*,H_1^*}\mathbf{P}_{23}\mathbf{L}_{H_4,H_3}\mathbf{P}_{23})\mathbf{P}_{23}\mathbf{L}_{H_2,H_1}\mathbf{P}_{23} \\ &= \mathbf{L}_{H_4^*,H_3^*}(\mathbf{N}_{H_2^*,H_1^*,H_4,H_3})\mathbf{P}_{23}\mathbf{L}_{H_2,H_1}\mathbf{P}_{23} \end{aligned} \ . \tag{7}$$

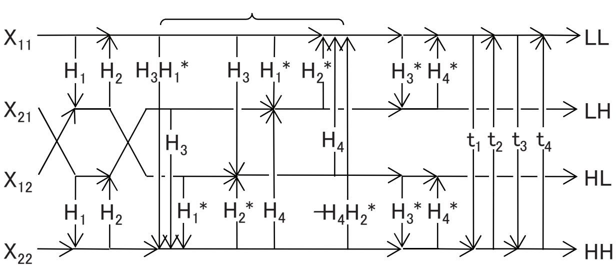
As a result, total number of the rounding operations is reduced from 32 to 16 as summarized in table 1. It is reduced by half comparing to the existing 2D DWT.



(a) Step 1; unify the scaling coefficients.



(b) Step 2; apply theorem 1.



(c) Step 3; apply theorem 2 to step 2.

Fig.3 Proposed non separable 2D DWT.

4. SIMULATION RESULTS

We investigate superiority of the proposed 2D DWT in Fig.3(c) to the existing 2D DWT in Fig.1(b) with 13 kinds of image signals listed in table 2.

4.1. Compatibility to the irreversible DWT

Table 3 summarizes three cases to be investigated in this report. In case 1, the forward reversible DWT $\mathbf{T}\circ$ is connected to the backward irreversible DWT $\mathbf{T}^{-1}$ and the final output signal is rounded into integer. In case 2, output of the forward irreversible DWT $\mathbf{T}$ is rounded and connected to the backward reversible DWT $\mathbf{T}^{-1}\circ$. In case 3, both of forward and backward transforms are reversible. In this case, lossless coding is available.

Fig.4(a),(b) summarize the PSNR of the mismatching error (compatibility) defined by

$$PSNR = -10\log_{10}\frac{E\left[trace\left(\mathbf{D}\mathbf{D}^T\right)\right]}{255^2} \tag{8}$$

where

$$\begin{cases} \mathbf{D} = R\left[\mathbf{T}^{-1}\mathbf{T}\circ\mathbf{X}\right] - \mathbf{X} & , \ case\ 1 \\ \mathbf{D} = \mathbf{T}^{-1}\circ R\left[\mathbf{T}\mathbf{X}\right] - \mathbf{X} & , \ case\ 2 \end{cases}$$

and $E[\ ]$ denotes ensemble average over all the pixel values in an image signal.

Table 1 The number of rounding operations.

	lifting	scaling	total
proposed	12	4	16 (50.0%)
existing [10]	16	16	32 (100%)

Table 2 List of evaluated image signals.

1.boat	2.girl	3.couple	4.cameraman	
5.text	6.bridge	7.airplane	8.lax	9.building
10.barbara	11.lenna	12.woman	13.lighthouse	

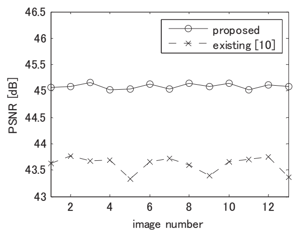
Table 3 Three cases to be evaluated.

	case 1	case 2	case 3
forward	reversible $\mathbf{T}\circ$	irreversible $\mathbf{T}$	reversible $\mathbf{T}\circ$
backward	irreversible $\mathbf{T}^{-1}$	reversible $\mathbf{T}^{-1}\circ$	reversible $\mathbf{T}^{-1}\circ$
output	$R[\mathbf{T}^{-1}\mathbf{T}\circ\mathbf{X}]$	$\mathbf{T}^{-1}\circ R[\mathbf{T}\mathbf{X}]$	$\mathbf{T}^{-1}\circ\mathbf{T}\circ\mathbf{X}$

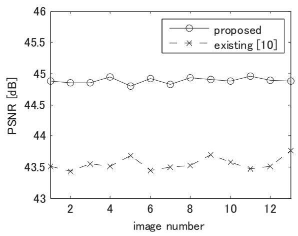
Table 4 Compatibility and lossless coding performance.

	compatibility PSNR [dB]		lossless coding entropy [bpp]
	case 1	case 2	case 3
proposed	45.09 (+1.25)	44.88 (+1.21)	5.10
existing [10]	43.83 (+0.00)	43.67 (+0.00)	5.11

Table 4 summarizes the PSNR averaged over all the image signals in table 2 for case 1 and case 2. It is confirmed that the proposed DWT improves the compatibility by 1.2 [dB]. Fig.6 illustrates example of the error in case 2 for 'Lenna'.

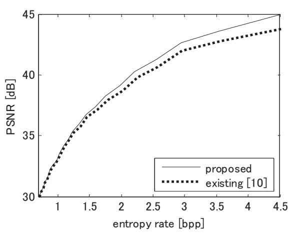


(a) case 1

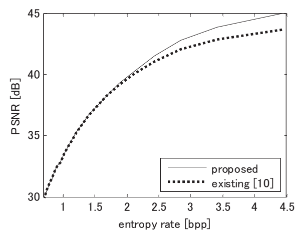


(b) case 2

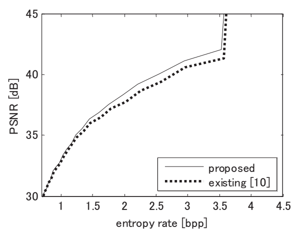
Fig.4 Compatibility to the irreversible DWT.



(a) case 1

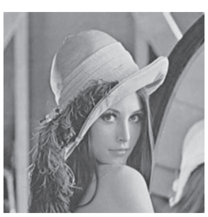


(b) case 2

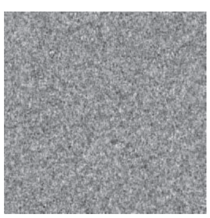


(c) case 3

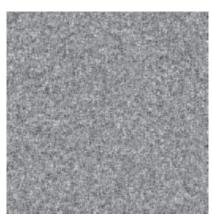
Fig.5 Lossy coding performance.



(a) original



(b) existing



(c) proposed

Fig.6 Original image and its error signals **D'**.

case 1; $\mathbf{D}' = (R[\mathbf{T}^{-1}\mathbf{T} \circ \mathbf{X}] - \mathbf{X}) * 16 + 128$.

4.2. Coding Performance

Lossless coding performance is summarized in case 3 in Table 4. The entropy rate by the EBCOT [1] is evaluated for 'Lenna' with 2 stage octave decomposition of the DWT. No significant difference is found in lossless coding. Fig.5(d) indicates lossy coding performance in the three cases where the bit truncation is applied for the rate control. Superiority in PSNR of the proposed DWT to the existing DWT is confirmed at high bit rate lossy coding.

5. CONCLUSIONS

In this report, we halved the number of the rounding operations of the existing reversible 2D DWT by factorizing the separable 2D transfer function of the 2D DWT into non separable 2D functions. It was found that the proposed 2D DWT reduces the rounding error and increases the PSNR by +1.2 [dB] in average over thirteen kinds of image signals. It was confirmed that our method improves compatibility to the irreversible 2D DWT in the JPEG 2000, maintaining its lossless coding performance.

6. REFERENCES

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