Methods for Avoiding the Checkerboard Distortion Caused by Rounding Error in Multirate System

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Abstract

We propose two methods for avoiding the checkerboard distortion which is caused by rounding error under finite word length. In the first method, the bit length of filter taps required for avoiding the checkerboard distortion is derived. In the second method, the cascade structure that consist of zero-hold kernel and time-invariant filter factorized from the filter with structure for avoiding the checkerboard distortion under linear conditions is proposed. It is demonstrated by simulations that we can avoid the checkerboard distortion by using these proposed methods.

keywords: checkerboard distortion, multirate system, finite word length, approximation error

1. Introduction

Multirate signal processing techniques has been applied to many areas such as filter banks, analog-to-digital, digital-to-analog conversion, image resolution conversion, and so on [1, 2]. The multirate system consists of interpolator and decimator, since these techniques are realized on sampling rate conversion. It is known that the checkerboard distortion occurs owing to periodic time-variant of the interpolator [4, 5]. On the other hand, several conditions for avoiding this checkerboard distortion has been also proven [4, 5]. However, these conditions are not necessarily approved in real systems because infinite word length is assumed in these conditions. In other words, these conditions are not guaranteed because of rounding error under finite word length, then there is a possibility that the checkerboard distortion occurs.

In this paper, two methods for avoiding the checkeboard distortion under finite word length are proposed. In the first method, the bit length of filter taps required for avoiding the checkerboard distortion is derived. In the second method, we propose the cascade structure that consist of zero-hold kernel and time-invariant filter factorized from the filter which has the structure for avoiding the checkerboard distortion under linear conditions. It is demonstrated by experiments of image resolution conversion in last section that we can avoid the checkerboard distortion by using these proposed methods.

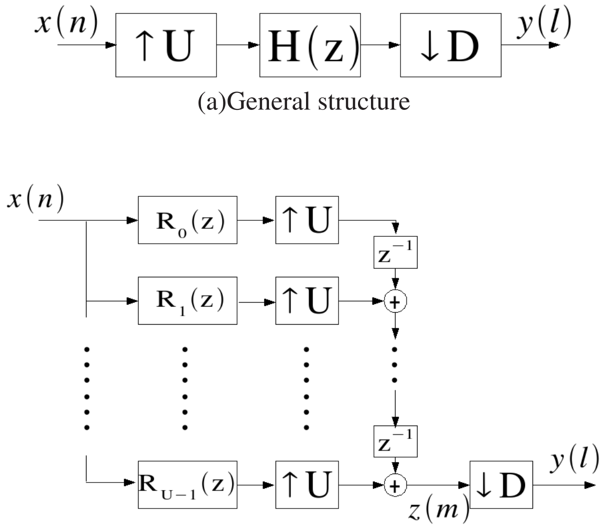


Figure 1: sampling rate converter.

2. Checkerboard distortion

In this section, firstly, sampling rate conversion is described. Secondly, we explain about cause of the checkboard distortion. Moreover, conditions for avoiding the checkboard distortion under linear systems is described.

2.1. Sampling rate conversion

The composition of sampling rate converter is shown in Fig.1(a), where U and D are mutually prime positive integers, $H(z)$ is a digital filter, $\uparrow U$ is up-sampler with the factor U, and $\downarrow D$ is down-sampler with the factor D. $H(z)$

is the transfer function of an FIR filter given as

$$H(z) = \sum_{n=-\infty}^{\infty} h(n) z^{-n}, \tag{1}$$

where $h(n)$ denotes the impulse response of $H(z)$.

Fig.1(a) can be equivalently represented as polyphase structure shown in Fig.1(b) [3–5]. $H(z)$ can be represented as

$$H(z) = \sum_{i=0}^{U-1} R_i(z^U) z^{-U-1-i}, \tag{2}$$

where

$$R_i(z) = \sum_{n=-\infty}^{\infty} h(Un - i - 1 + U) z^{-n}. \tag{3}$$

$R_i(z)$ is a polyphase filter of $H(z)$.

2.2. Definition of checkerboard distortion

In Fig.1(b), if the input signal $x(n)$ is the unit step signal $u(n)$ and besides n is large enough, $z(m)$ becomes steady-state value given as

$$s(m) = \begin{cases} R_0(1), & (m = Un + (U-1)), \\ R_1(1), & (m = Un + (U-2)), \\ \vdots & \\ R_{U-1}(1), & (m = Un), \end{cases} \tag{4}$$

where $R_i(1)$ denotes DC gain of the polyphase filter $R_i(z)$. Consequently, the steady-state value of output signal $y(l)$ is given as

$$y(l) = s(Dm). \tag{5}$$

Because each $R_i(1)$ does not have generally same value, $s(m)$ is not constant and has the period U. In consequence, $y(l)$ has also the period U. The periodic artifact caused by this periodic step response is called the checkerboard distortion [4, 5].

2.3. Conditions for avoiding the checkerboard distortion under linear condition

Several conditions for avoiding the checkerboard distortion in under linear condition has been proven [4, 5].

Theorem 1: A necessary and sufficient condition for avoiding the checkerboard distortion is given as

$$R_0(1) = R_1(1) = \cdots = R_{U-1}(1). \tag{6}$$

Theorem 2: If $H(z)$ can be factorized as Eq.(7), we can avoid the checkerboard distortion.

$$H(z) = H_0(z) P(z). \tag{7}$$

where

$$H_0(z) = \sum_{i=0}^{U-1} z^{-i}, \tag{8}$$

$H_0(z)$ is proper for interpolation kernel of zero-order hold. When $H(z)$ has zero point at z shown in Eq.(9), Eq.(7) is satisfied.

$$z = e^{j\omega} \mid_{\omega = \frac{2\pi m}{U}}, m = 1, 2, \cdots, U-1, \tag{9}$$

In addition, this theorem1 and theorem2 is equivalent conditions.

3. Checkerboard distortion under finite word length

The necessary and sufficient conditions described in section 2.3 premises that filter taps don't have rounding error caused by quantization under finite word length. Therefore, in real system, a method unaffected by rounding error is needed to build the multirate system without the checkerboard distortion. In this section, firstly, quantization used in this paper is defined. Secondly, we consider the checkerboard distortion under finite word length, and present two methods for avoiding it.

3.1. Definition of quantization

In this paper, we consider that the signal value is expressed by fixed point number defined as

$$\begin{aligned} & x = \sum_{p=-F}^{I-1} b_p 2^p, \qquad b_p \in 0, 1, \\ & I \geq 1, F \geq 0, \qquad I, F \in \mathbf{Z}, \end{aligned} \tag{10}$$

where I denotes number of bits of integral parts with sign bit, and F denotes number of bits of fractional parts. The sign bit is assigned to MSB. We define quantization of a certain value x as Eq.(11).

$$\begin{aligned} & O[x] = \lfloor x' \rfloor = x' - (x' \mod 1), \\ & x' = x + 2^{-1}. \end{aligned} \tag{11}$$

In addition, after this, we normalize a certain value W such as the input signal and filter taps by bit shift, and we proceed to a discussion on the assumption that W_{nor} is more than -1 but less than 1, where W_{nor} denotes normalized W. In case a certain value is quantized at V bits with sign bit, Eq.(12) is employed.

$$Q_V[W_{nor}] = O[W_{nor} 2^{(V-1)}] 2^{-(V-1)}. \tag{12}$$

3.2. The checkerboard distortion caused by rounding error

In real systems, the digital filter $H(z)$ is quantized because of constraint of finite word length. The polyphase structure of this quantized filter is defined as

$$R'_i(z) = \sum_{n=-\infty}^{\infty} h'(Un - i - 1 + U) z^{-n}, \tag{13}$$

where

$$\begin{aligned} & h'(Un - i - 1 + U) \\ & \quad = O(h(Un - i - 1 + U) 2^{(B-1)}) 2^{-(B-1)}. \end{aligned} \tag{14}$$

and B denotes number of bits assigned to filter taps. Even if $H(z)$ is designed with the condition for avoiding the checkerboard distortion under linear condition, there is a possibility that steady-state value of the step response has periodicity because DC gain of polyphase filter $R'_i(z)$ is not constant owing to rounding error caused by quantization under finite word length. As a result, the checkerboard distortion may occur.

3.3. Methods for avoiding the checkerboard distortion

method 1: calculating the bit length of filter taps required for avoiding the checkerboard distortion

In general, the output value of sampling rate converter is quantized at bit length of output signal. Thus, we can consider that the checkerboard distortion disappear if steady-state value of conclusive step response obtained after this quantization is constant, even if the checkerboard distortion occurs by rounding error. In addition, when bit length of the input signal is L bits, conceivable input step signals is given as

$$\begin{aligned} &x_K(n) = K2^{-(L-1)}u(n), \\ &K = -2^{(L-1)}, -2^{(L-1)}+1, \cdots, 2^{(L-1)}-1. \end{aligned} \tag{15}$$

In section 2.2, we described the checkerboard distortion as periodical oscillation of unit step response. However, it is not enough to handle unit gain only if we cannot suppose linearity. Therefore, after this, we consider it as periodicity of a step response of $x_K(n)$.

From here onwards, Eq.(16) is derived. The chekerboard distortion does not occur if this inequations are satisfied.

$$\begin{cases} \frac{(-2^{-V}+\Delta f'_{i'})}{|x_K(1)|} \le e_{i'} < \frac{(2^{-V}+\Delta f'_{i'})}{|x_K(1)|}, \\ \qquad\qquad (e_{i'}x_K(1) > 0), \\ \frac{(-2^{-V}-\Delta f'_{i'})}{|x_K(1)|} \le e_{i'} < \frac{(2^{-V}-\Delta f'_{i'})}{|x_K(1)|}, \\ \qquad\qquad (e_{i'}x_K(1) < 0), \end{cases} \tag{16}$$

where

$$\Delta f'_{i'} = Q_V[f'_{i'}] - f'_{i'}, \;\; i' = 0, 1, \cdots, U-2, \tag{17}$$

$$f'_i = R'_i(1)x_K(1), \;\; i = 0, 1, \cdots, U-1, \tag{18}$$

$$e_{i'} = R'_{i'+1}(1) - R'_{i'}(1), \tag{19}$$

and V denotes a word length of output signal. The details of Eq.(16) are described in reference [6]. However, this reference has shown the fact that it depends on not only bit length of the filter but also such as value of input signal and filter taps whether the checkerboard distortion occurs or not.

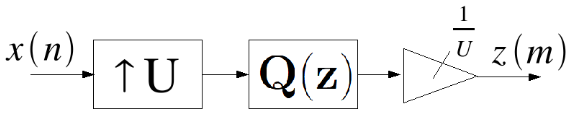


Figure 2: The interpolator designed from Eq.(20).

method 2: using the cascade structure

Alternatively, we present the method which does not depend on any situations for avoiding the checkerboard distortion. A digital filter designed from theorem 2 can be factorized as shown by Eq.(7). Thus, the interpolator can consist of zero-order hold and time-invariant filter $P(z)$. The checkerboard distortion can be certainly avoided when this zero-order hold and time-invariant filter are respectively quantized, because zero-order hold is unaffected by quantization, in addition, time-invariance of quantized filter $P'(z)$ is retained. However, the filtering of $H_0(z)$ must be implemented before $P'(z)$.

4. Simulation

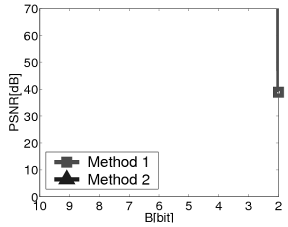
It is presented by image resolution conversion that the checkerboard distortion caused by rounding error affect the image quality, in addition, we can avoid the checkerboard distortion by using the proposed methods.

The image quality was evaluated by calculating PSNR between ideal condition and interpolation results obtained by using method 1 and method 2. The ideal condition is results obtained by using the filter with enough word length. The input bit length L and output bit length V were respectively 8, and the test image was “lenna". The bit length of filter taps B were changed from 10 to 2. Linear, B-spline, and cubic convolusion was respectively used for interpolation function. The filter of linear interpolation can be represented as

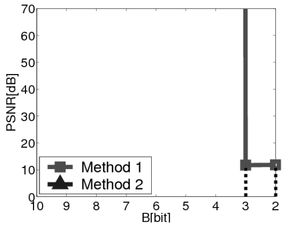
$$H(z) = \frac{1}{U}Q(z). \tag{20}$$

Thus, the multiplication $1/U$ was implemented behind interpolator shown in Fig.2. Similarly, since B-spline and cubic convolusion can be factorized $H(z)$ into $1/(6U^3)$ and the other, $1/(6U^3)$ was implemented at the last. Furthermore, filter gain may be not 1 because of quantization. As a result, even though the chekerboard distortion is avoided, there is a possibility that PSNR decreases. In this paper, we want to consider the relation between the checkerboard distortion and image quality. Thus, the output signal was compensated by multiplying the output signal by inverse of the filter gain.

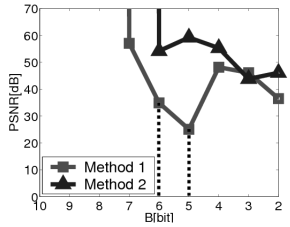
The results of interpolation is shown in Fig.3. The dashed line was drawn on the word length that the checkerboard distortion had occurred. In method 1, when the checkerboard distortion occurred, we can recognize that PSNR decreases significantly. By contrast, in method 2, because the checkerboard distortion don’t occur, high PSNR



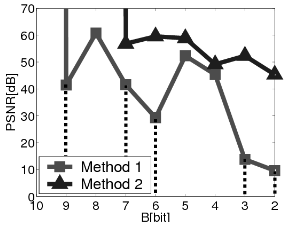
(a)Linear, $U = 3$.



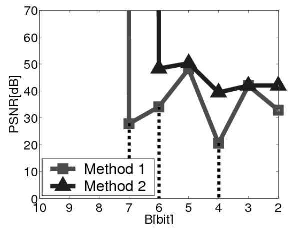
(b)Linear, $U = 4$.



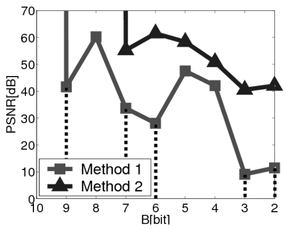
(c)B-spline, $U = 3$.



(d)B-spline, $U = 4$.



(e)Cubic convolusion, $U = 3$.



(f)Cubic convolusion, $U = 4$.

Figure 3: The results of interpolation.

(a)method 1

(b)method 2

Figure 4: The part of interpolation results. Cubic convolusion, $B = 6$.

is generally obtained. Fig.4(a) is the image with checkerboard distortion, which was interpolated threefold in method 1. Cubic convolusion was used for interpolation function, additionally $B = 6$. In contrast, Fig.4(b) is similarly the result interpolated in method 2. This image don't have the checkerboard distortion. In these results, if the method 2 is employed, we can certainly avoid the chekerboard distortion and obtain the result close to ideal condition.

5. Conclusion

The checkerboard distortion caused by rounding error under finite word length was discussed. The two methods for avoiding the checkerboard distortion under finite word length were proposed. In the first method, the bit length of filter taps required for avoiding the checkerboard distortion was derived. In the second method, the cascade structure that consist of zero-hold kernel and time-invariant filter factorized from the filter with structure for avoiding the checkerboard distortion under linear conditions was proposed. Especially, the second method is uninfluenced by such as bit length of filter taps and input value. It was presented by image resolution conversion that we can avoid the checkerboard distortion by using the proposed methods.

References

[1] A. V. Oppenheim, R. W. Schafer, and J. R. Buck, Discrete-time Signal Processing, Pearson Education, 1999.

[2] F. Harris, Multirate Signal Processing for Communication Systems, Prentice Hall, 2004.

[3] S. Muramatsu, A. Yamada and H. Kiya, "A Design Method of Multidimensional Linear-Phase Paraunitary Filter Banks with a Lattice Structure", IEEE Trans. Signal Processing., Vol. 47, no. 3, pp. 690-700, Mar, 1999.

[4] T. Tamura, M. Kato, T. Yoshida and A. Nishihara, "Design of Checkerboard-Distortion-Free Multidimensional Multirate Filters", IEICE Trans. Fundamentals., Vol. E81-A, no. 8, pp. 1598-1606, Aug, 1998.

[5] Y. Harada, S. Muramatsu and H. Kiya, "Multidimensional Multirate Filter and Filter Bank without Checkerboard Effect", IEICE Trans. Fundamentals., Vol. E81-A, no. 8, pp. 1607-1615, Aug, 1998.

[6] H. Iwai, M. Iwahashi and H. Kiya, "Conditions for Avoiding the Checkerboard Distortion in Multirate System with Finite Word Length Approximation", Proc. 23rd SIP Symposium., Nov, 2008.