# Word Length Assignment of DC Lossless DWT

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***Abstract*— This report investigates the minimum word length of coefficients and signals such that a lifting structured discrete wavelet transform (DWT) becomes lossless for a constant valued input signal (DC signal). It guarantees that the output signal contains no error in spite of rounding of coefficients and signals. This DC lossless condition is a necessary condition for the regularity which has been analyzed to improve performance of a transform. When the regularity is not satisfied, the DWT has checker board artifact observed in a reconstructed signal and DC leakage which decreases the coding gain. We derive the condition utilizing mutual affect between rounding of signals and that of coefficients. It is confirmed that the minimum word length under the condition derived by our analysis is shorter than that determined by a conventional analysis. It contributes to build a low complexity DWT under the condition.**

## I. INTRODUCTION

In this report, we investigate the minimum word length of coefficient values and that of signal values such that the DWT becomes lossless for a constant valued input signal (DC signal). Under this DC lossless condition, it is guaranteed that the output signal contains no error in spite of rounding of coefficients and signals inside the DWT.

In case of the 5-3 DWT in JPEG 2000 for lossless coding, benefiting from its lifting structure [1-3], lossless reconstruction of any signal is guaranteed even though signals and coefficients are rounded. On the contrary, it does not hold good for the 9-7 DWT because of scaling for adjustment of DC gain of a low pass filter in the forward DWT [4]. However, we have pointed out that it became possible to be lossless for a DC signal under a certain condition on word length of coefficients and signals [5].

This DC lossless condition is a necessary condition for the regularity which has been analyzed by numerous researchers to improve coding performance of a transform [6-8]. When the regularity is not satisfied, the DWT has the checker board artifact which is observed in a reconstructed signal as unnecessary high frequency noise [6]. It also brings about the DC leakage which decreases the coding gain.

The regularity has been structurally guaranteed for a quadrature mirror filter bank [6], a bi-orthogonal filter bank [7] and the DCT [8] respectively. However, since these previous methods are based on the lattice structure, these are not directly applicable to the lifting structure of the 9-7 DWT. Beside these relations to the regularity, the DC lossless condition itself is also considered to be useful for white balancing of a video system in which the DC signal is used as a test input for calibration [9].

This report investigates the minimum word length of the 9-7 DWT under the DC lossless condition. We have already analyzed the DC lossless condition utilizing mutual effect between rounding of signals and that of coefficients [10]. In this report, we assign the optimum word length to each coefficient, introducing tolerance for errors as a parameter to simultaneously control both of word length of signals and that of coefficients.

## II. WORD LENGTH AND DC LOSSLESS

### A. Word Length and Rounding Error

Assuming the fixed point binary representation of signal values and coefficient values [4], we use the rounding operation defined as

$$\begin{cases} R_0[s] = \lfloor s' \rfloor = s' - (s' \mod 1), \ s' = s + 2^{-1}, \\ R_{F_S}[s] = R_0[s2^{F_S}]2^{-F_S}, \ 0 \le F_S \in \mathbf{Z}. \end{cases} \tag{1}$$

The operation $R_{F_S}[s]$ shortens the fraction part of word length of a value $s$ into $F_S$ [bit]. It also generates the error:

$$\Delta_{F_S}[s] = s - R_{F_S}[s]. \tag{2}$$

Denoting the integer part as $I_S$ [bit], total word length $W_S$ [bit] of a signal $s$ is defined as

$$W_S = I_S + F_S + 1 \tag{3}$$

including 1 [bit] for the sign part. Similarly, total word length $W_C$ [bit] of a coefficient $c$ is defined as

$$W_C = I_C + F_C + 1 \ . \tag{4}$$

Especially, we utilize properties of the rounding operation and its errors [10,11]:

$$\left|\Delta_{F_S}[s]\right| \le 2^{-1-F_S} \tag{5}$$

$$R_{F_S}[s]2^{F_S} = p \in \mathbf{Z} \tag{6}$$

$$R_{F_S}[s]2^{F_S} = p \ \Leftrightarrow \ |s| \le (p + 2^{-1})2^{-F_S} \tag{7}$$

$$s2^{F_S} \in \mathbf{Z} \Rightarrow R_{F_S}[s+t] = s + R_{F_S}[t] \tag{8}$$

which hold for any real number $s$ and $t$. In Eq.(6) and (7), $p$ is an integer determined by $s$ under a given integer $F_s$.

### B. Definition of DC Lossless

A forward DWT (two channel 1D filter bank) splits an input signal $x(n)$ with $F_X$ [bit] fraction part into a low band signal $y_1(m)$ and a high band signal $y_1(m)$. Fraction part of these band signals are rounded to $F_B$ [bit] at output. These are



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synthesized by the backward DWT and rounded to $F_X$ [bit] at output to reconstruct a signal $w(n)$. Inside the DWT, signals are multiplied by coefficients with $F_C$ [bit] and rounded into $F_S$ [bit]. Our concern is to find the minimum of $F_C$ and $F_S$ for a given $F_X$ such that the 9-7 DWT becomes DC lossless.

The DC lossless is defined as the conjunction of the following two propositions:

$$\forall n \in \mathrm{N}, m \in \mathrm{M}\ (\ x(n) = d \ \rightarrow \ y_2(m) = 0\ ) \tag{9}$$

$$\forall n \in \mathrm{N}\ (\ x(n) = d \ \rightarrow \ w(n) = d\ ) \tag{10}$$

for a given constant value $d$ with $F_X$ [bit] fraction part. When the proposition in Eq.(9) holds, the DWT has no DC leakage for the DC input signal with value $d$. Similarly, when the proposition in Eq.(10) is true, the reconstructed signal $w(n)$ contains no checker board artifact for the DC input signal.

## III. DC Lossless Condition on Word Length

### A. Model for Error Analysis

Fig.1(a) illustrates a multiplier in the DWT circuit. An input value $s$ has $F_S$ [bit] fraction part and multiplied by a coefficient $c'$. The coefficient is originally designed as a real number $c$. It is rounded to a rational number $c'$ in implementation. Fig.3(b) illustrates our model for theoretical analysis. The output value is described as

$$\begin{cases} s' = cs + e \\ e = R_{F_S}\left[\Delta_{F_S}[cs] - \Delta_{F_C}[c]s\right] - \Delta_{F_S}[cs] \end{cases}. \tag{11}$$

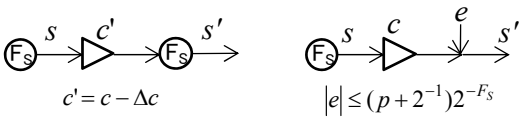
In this model, both of coefficient error and signal error are unified to the error $e$. Utilizing Eq.(5) and (7), its maximum value is described as

$$|e| \le (p + 2^{-1})2^{-F_S}\ . \tag{12}$$

Note that the parameter $p$ to control word length of a coefficient $c$ is included. It is equivalent to

$$2^{-F_C + F_S + I_S - 1} \le p\ . \tag{13}$$

From this inequality, it becomes possible to consider mutual effect of the coefficient error and the signal error [10].




(a) Multiplier. (b) Model for analysis.

Fig. 1 A multiplier and its models for analysis.

### B. DC Equivalent Circuit

When the input signal is restricted to a DC signal, $x(n)$ can be described as a scalar $x$ independent of $n$. The delay $z^{-1}$ can be treated as 1 and $(1+z^{-1})$ can be replaced by 2. As a result, we can use equivalent circuits for a DC input signal in Fig.2 to derive the condition [10].

In Fig.2(a), a scalar $x$ with $F_X$ [bit] fraction part is multiplied by a rational number $c_i$ , $i \in \mathrm{I}$ and rounded to $F_S$ [bit]. The signals are rounded to $F_B$ [bit] at its output to produce two scalars $[y_1\ y_2]$. The unified errors inside the circuit are described as

$$\begin{cases} e_i = R_{F_S}\left[\Delta_{F_S}[c_i s_i] - \Delta_{F_C}[c_i]s_i\right] - \Delta_{F_S}[c_i s_i] \\ f_i = R_{F_S}\left[\Delta_{F_S}[c_i t_i] - \Delta_{F_C}[c_i]t_i\right] - \Delta_{F_S}[c_i t_i] + f_i'' \end{cases} \tag{14}$$

where

$$\begin{bmatrix} s_1 & s_3 & s_5 \\ s_2 & s_4 & s_6 \end{bmatrix} = \begin{bmatrix} 2x & 2(x + s'_2) & s_4 \\ 2(x + s'_1) & 2(s_2 + s'_3) & s_3 + s'_4 \end{bmatrix}$$

$$\begin{bmatrix} t_1 & t_3 & t_5 \\ t_2 & t_4 & t_6 \end{bmatrix} = \begin{bmatrix} 2(t_3 - t'_2) & 2(t'_5 - t'_4) & y_1 \\ 2(t_4 - t'_3) & 2t'_6 & y_2 \end{bmatrix}$$

and

$$\begin{cases} s'_i = c_i s_i + e_i = R_{F_S}[c_i s_i] \\ t'_i = c_i t_i + f_i = R_{F_S}[c_i t_i]. \end{cases}$$

Similarly to Eq.(12), these errors are described with the parameters $p_i$ and $q_i$ to control word length of coefficients as

$$\begin{cases} |e_i| \le (p_i + 2^{-1})2^{-F_S} \\ |f_i| \le (q_i + 2^{-1})2^{-F_S} \end{cases} \tag{15}$$

for a given word length $F_s$ [bit] of signals.

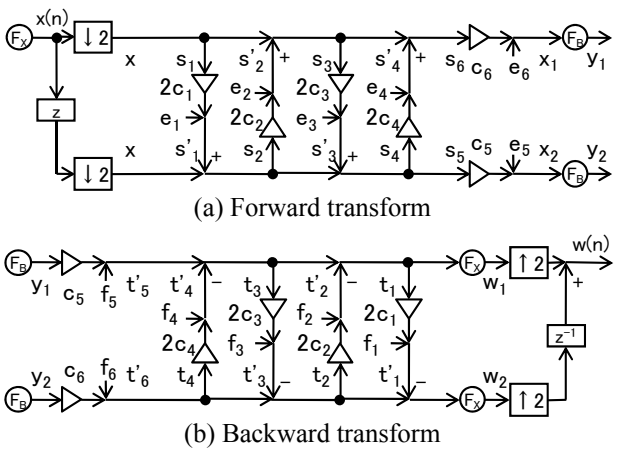



Fig. 2 Equivalent circuits of the DWT for a DC input.

### C. Nullification of Accumulated Errors

In Fig.2(a), the unified errors in Eq.(15) are propagated and accumulated inside the circuit. When the accumulated errors are nullified by the rounding at output of the forward transform, Eq.(9) is satisfied. In the figure, $\mathbf{Y}_{12}=[y_1\ y_2]^T$ and $\mathbf{W}_{12}=[w_1\ w_2]^T$ are described as

$$\begin{cases} \mathbf{Y}_{12} = R_{F_B}\left[(\mathbf{H}_{e1}\mathbf{E}_1 + \mathbf{H}_{e2}\mathbf{E}_2) + \mathbf{K}\mathbf{H}_{4321}x\right] \\ \mathbf{W}_{12} = R_{F_X}\left[\ (\mathbf{H}_{e3}\mathbf{E}_3 + \mathbf{H}_{e4}\mathbf{E}_4) + (\mathbf{K}\mathbf{H}_{4321})^{-1}\mathbf{Y}_{12}\right] \end{cases} \tag{16}$$

where the errors are

$$\begin{cases}\mathbf{E}_1=[e_6 \quad e_4 \quad e_2]^T \\ \mathbf{E}_2=[e_5 \quad e_3 \quad e_1]^T\end{cases} \quad \begin{cases}\mathbf{E}_3=[f_2 \quad f_4 \quad f_5]^T \\ \mathbf{E}_4=[f_1 \quad f_3 \quad f_6]^T\end{cases}$$

and transfer function to the errors are

$$\begin{cases}\mathbf{H}_{e1}=[\mathbf{I}_U \quad \mathbf{K}\mathbf{I}_U \quad \mathbf{K}\mathbf{H}_{43}\mathbf{I}_U] \\ \mathbf{H}_{e2}=[\mathbf{I}_L \quad \mathbf{K}\mathbf{H}_4\mathbf{I}_L \quad \mathbf{K}\mathbf{H}_{432}\mathbf{I}_L]\end{cases} \quad \begin{cases}\mathbf{H}_{e3}=-[\mathbf{H}_1^{-1}\mathbf{I}_U \quad \mathbf{H}_{123}^{-1}\mathbf{I}_U \quad \mathbf{H}_{1234}^{-1}\mathbf{I}_U] \\ \mathbf{H}_{e4}=-[\mathbf{I}_L \quad \mathbf{H}_{12}^{-1}\mathbf{I}_L \quad \mathbf{H}_{1234}^{-1}\mathbf{I}_L]\end{cases}$$

In the equations above, we used the notations:

$$\mathbf{H}_{i\in\{1,3\}}=\begin{bmatrix}1 & 0\\ 2c_i & 1\end{bmatrix},\ \mathbf{H}_{j\in\{2,4\}}=\begin{bmatrix}1 & 2c_j\\ 0 & 1\end{bmatrix},\ \mathbf{K}=\begin{bmatrix}c_6 & 0\\ 0 & c_5\end{bmatrix},$$

$$\mathbf{H}_{43\cdots i}=\mathbf{H}_4\mathbf{H}_3\cdots\mathbf{H}_i,\quad \mathbf{H}_{12\cdots i}^{-1}=\mathbf{H}_1^{-1}\mathbf{H}_2^{-1}\cdots\mathbf{H}_i^{-1},$$

$$\mathbf{I}_U=[1 \quad 0]^T,\ \mathbf{I}_L=[0 \quad 1]^T,\ \mathbf{I}_{UL}=\mathbf{I}_U+\mathbf{I}_L.$$

Subtracting the ideal output from the output in Eq.(16), we have the accumulated errors as

$$\begin{bmatrix}\mathbf{E}_{y12}\\ \mathbf{E}_{w12}\end{bmatrix}=\begin{bmatrix}\mathbf{Y}_{12}\\ \mathbf{W}_{12}\end{bmatrix}-\begin{bmatrix}\mathbf{I}_U\\ \mathbf{I}_{UL}\end{bmatrix}x\ . \tag{17}$$

Finally, using the property in Eq.(8), we have

$$\begin{bmatrix}\mathbf{E}_{y12}\\ \mathbf{E}_{w12}\end{bmatrix}=\begin{bmatrix}R_{F_B}\left[(\mathbf{H}_{e1}\mathbf{E}_1+\mathbf{H}_{e2}\mathbf{E}_2)\right]\\ R_{F_X}\left[(\mathbf{H}_{e3}\mathbf{E}_3+\mathbf{H}_{e4}\mathbf{E}_4)\right]\end{bmatrix}. \tag{18}$$

Applying Eq.(7) with $p$=0, it becomes clear that when the conditions:

$$\begin{cases}\left|\mathbf{H}_{e1}\mathbf{E}_1+\mathbf{H}_{e2}\mathbf{E}_2\right|\le\mathbf{I}_{UL}2^{-1-F_B}\\ \left|\mathbf{H}_{e3}\mathbf{E}_3+\mathbf{H}_{e4}\mathbf{E}_4\right|\le\mathbf{I}_{UL}2^{-1-F_X}\end{cases} \tag{19}$$

are satisfied, the accumulated errors are nullified by the rounding operations at output of the transforms.

### *D. Critical Condition on Word Length*

Based on discussions above, we derive the condition on word length of coefficients and signals such that the condition in Eq.(19) is satisfied. Since the unified errors in $\mathbf{E}_1$, $\mathbf{E}_2$, $\mathbf{E}_3$ and $\mathbf{E}_4$ have the maximum in Eq.(15) described with the parameters $p_i$ and $q_i$, the DC lossless condition is also described with the parameters by substituting

$$\begin{cases}\mathbf{E}_1=\left([p_6 \quad p_4 \quad p_2]^T+\mathbf{I}_3\cdot 2^{-1}\right)2^{-F_S}\\ \mathbf{E}_2=\left([p_5 \quad p_3 \quad p_1]^T+\mathbf{I}_3\cdot 2^{-1}\right)2^{-F_S}\\ \mathbf{E}_3=\left([q_2 \quad q_4 \quad q_5]^T+\mathbf{I}_3\cdot 2^{-1}\right)2^{-F_S}\\ \mathbf{E}_4=\left([q_1 \quad q_3 \quad q_6]^T+\mathbf{I}_3\cdot 2^{-1}\right)2^{-F_S}\end{cases} \tag{20}$$

into Eq.(19) where $\mathbf{I}_3=[1\ 1\ 1]^T$. This is the condition we derived based on the model in III.*A*.

In IV, we investigate the minimum $F_{Ci}$ [bit] of a coefficient $c_i$, $i\in$ I under the DC lossless condition for a given DC value $x$ and $F_S$ [bit] of signals inside the DWT.

### *E. Sufficient Condition on Word Length*

In case of all the parameter in Eq.(20) are given as $p_i=q_i=p$ and $F_{Ci}=F_C$ for $\forall\, i\in$ I, the condition in Eq.(19) becomes

$$\begin{cases}\left(\left\|\mathbf{H}_{e1}\right\|_{L^1}+\left\|\mathbf{H}_{e2}\right\|_{L^1}\right)\cdot 2^{-F_S}(p+2^{-1})\le\mathbf{I}_{UL}2^{-1-F_B}\\ \left(\left\|\mathbf{H}_{e3}\right\|_{L^1}+\left\|\mathbf{H}_{e4}\right\|_{L^1}\right)\cdot 2^{-F_S}(p+2^{-1})\le\mathbf{I}_{UL}2^{-1-F_X}\end{cases} \tag{21}$$

where $\|\mathbf{H}\|_{L^1}$ denotes a column vector whose component is a sum of absolute value of all components in each row. Substituting coefficients of the DWT into Eq.(21), we have

$$p\le 2^{-1+F_S-G_E}-2^{-1},\quad G_E=2.66\quad[\text{bit}] \tag{22}$$

for $F_X=F_B=0$. As a result, the DC lossless condition on the word length is given as

$$-\log_2(2^{-\Delta W_C}+2^{-\Delta W_S})\ge G_E \tag{23}$$

where

$$[\Delta W_C \quad \Delta W_S]=[F_C-I_S \quad F_S]$$

and $G_E$ is the lower bound. This means a sufficient condition for the DC lossless. Since it is too strict, the word length under this condition is redundant. Unlike this sufficient condition, our critical condition given as Eq.(19) under Eq.(20) determines the word length minimum and necessary for the DC lossless.

## IV. Simulation Results

### *A. Word Length under the Conditions*

In Fig.3, the sufficient condition in III.*E* is plotted as 'solid' line for any 8 bit integer $x$ ($W_X$=$I_X$=8, $F_X$=0). The optimum word length under this condition is ($F_S$,$F_C$)=(4,12) as indicated as 'Existing'. It guarantees the DC lossless, however the condition is too strict. Therefore the word length is redundant and there is room for further reduction.

In Fig.3, a cross '×' indicates a pair ($F_S$,$F_C$) which satisfies the critical condition in III.*D*. The minimum of $F_C$ for each $F_S$ is indicated as a 'broken' line. It is clear that the word length derived by our critical condition is shorter than that by the sufficient condition based on a conventional approach.

Table I summarizes the minimum word length under the conditions. It is observed that the word length can be reduced by limiting input DC signals to a specific value. Unlike the conventional analysis, our analysis gives the minimum word length shorter than that determined by the sufficient condition for each of input DC values.

Table I The minimum word length for a specific value for white balancing of a video system.

| condition | sufficient | critical | | |
|---|---|---|---|---|
| input DC value | any $x_{in}\in[0,2^8)$ | | $x_{in}$= 16 (black) | $x_{in}$=235 (white) |
| $F_S$ (signal) | 4 | | 2 | 3 |
| $F_C$ (coefficient) | 12 | | 12 | 9 | 9 |

$$x_{in}=x-2^{I_X-1}$$

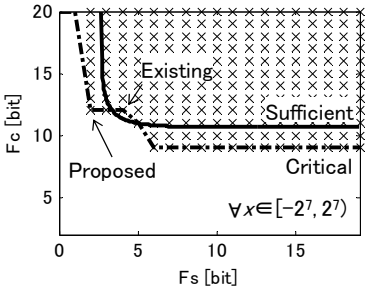


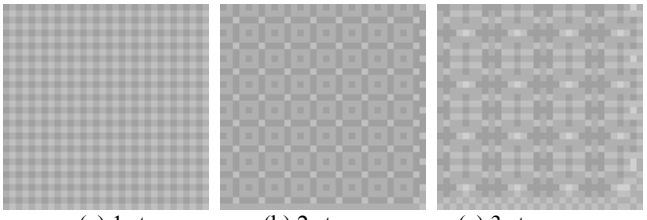

Fig.3 Word length under the conditions. " × " indicates ($F_S$, $F_C$) such that the DWT becomes DC lossless.

Table II Tolerance parameters for $x_{in}$=16 and $F_S$=2.

| $F_C$ | $p_1$ | $p_2$ | $p_3$ | $p_4$ | $p_5$ | $p_6$ | $y_1$-$x$ | $y_2$ |
|---|---|---|---|---|---|---|---|---|
| 9 | **0** | **0** | **0** | **0** | **0** | **1** | **0** | **0** |
| 8 | **0** | -3 | **0** | **0** | **0** | 0 | -1 | -1 |
| 7 | **0** | -3 | **0** | **0** | **0** | 0 | -1 | -1 |
| 6 | 7 | 12 | -8 | **0** | **0** | 2 | 6 | 6 |
| 5 | 7 | -18 | 10 | **0** | 1 | 0 | -7 | -5 |
| 4 | -21 | -18 | 9 | **0** | -2 | **1** | -10 | -12 |
| 3 | 35 | 107 | 7 | 25 | 9 | -28 | 63 | 70 |

Table III The minimum word length of coefficients of the 9-7 DWT for specific values.

| input values | | signals | coefficients | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | $F_S$ | $F_{C1}$ | $F_{C2}$ | $F_{C3}$ | $F_{C4}$ | $F_{C5}$ | $F_{C6}$ | *ave.* |
| for-ward | $x_{in}$= 16 (B) | 2 | 7 | 9 | 7 | 4 | 6 | 4 | **6.17** |
| | $x_{in}$=235 (W) | 3 | 9 | 7 | 9 | 9 | 1 | 4 | **6.50** |
| back ward | $x_{in}$= 16 (B) | 2 | 9 | 9 | 7 | 0 | 8 | 0 | **5.50** |
| | $x_{in}$=235 (W) | 3 | 9 | 9 | 7 | 0 | 8 | 0 | **5.50** |



(a) 1 stage (b) 2 stages (c) 3 stages

Fig.4 Example of reconstructed images for $128^2$ pixel DC input image with $x$=10. Intensity is multiplied by 16.

### *B. Optimum Word Length Assignment*

Since we described tolerance for the unified errors as parameters $p_i$ and $q_i$ in Eq.(15), it becomes possible to simultaneously control both of word length of signals and that of coefficients.

Table II summarizes these parameters for an input value 16 and the word length of signals at $F_S$=2 [bit] as an example. It indicates that [$p_1$ $p_2$ ⋯ $p_6$] are [0 0 0 0 0 1] for $F_C$=9 [bit]. In this case, all the coefficients $c_i$ in the forward transform have the same length. Note that the parameter $p_1$ is the same for $F_C$=9, 8 and 7 [bit]. It means that word length of the coefficient $c_1$ can be reduced from 9 to 7 [bit] without any influence to the errors. As a result, [$F_{C1}$ $F_{C2}$⋯$F_{C6}$] can be reduced from [9 9 9 9 9 9] to [7 9 7 4 6 4].

Table III summarizes results of this optimum word length assignment for the DWT. Comparing to table I, it is observed that word length of coefficients is reduced from 9.00 [bit] to 6.17 [bit] on average for an input value $x_{in}$=16.

Fig.4 illustrates image signals reconstructed by the DWT which does not satisfy the DC lossless condition. It demonstrates the checker board artifact for reference. A DWT with the coefficients under the condition described in this report does not have such artifacts.

## V. CONCLUSIONS

Utilizing the nullification of the accumulated errors, we theoretically derived a condition on word length of signals and coefficients such that the 9-7 DWT of JPEG 2000 becomes lossless for a DC input signal. It was confirmed that the minimum word length derived by our 'critical' condition was shorter than that determined by a conventional 'sufficient' condition. Analysis in this report contributes to build a low complexity DC lossless DWT.

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