Regularity Guaranteeing Implementation of DWT Designed under Lifting Structure

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***Abstract*- This report introduces a regularity guaranteeing section in implementation of a discrete wavelet transform (DWT) to reduce an artifact, which is due to finite word length expression of coefficients, in a reconstructed signal. The DWT is initially designed to satisfy the regularity under long enough word length of coefficient values. However these are truncated to shorter length in implementation to reduce computational cost. Then some errors are observed in a reconstructed signal. In this report, we implement the lifting DWT in a non-lifting form, instead of the lifting form. It includes a regularity guaranteeing section so that the DWT always satisfy the regularity in spite of shortening word length. It was confirmed that an oscillation in a reconstructed signal was completely suppressed.**

I. INTRODUCTION

Over the past few decades, the lifting structure has been utilized to design a perfect reconstruction filter bank equivalent to the bi-orthogonal discrete wavelet transform (DWT) [1,2]. In case of the 5-3 DWT in JPEG 2000 for lossless coding [3], benefiting from its lifting form, lossless reconstruction of any input signal is guaranteed even though signal values and coefficient values are truncated.

On the contrary, it does not hold good for the 9-7 DWT in JPEG 2000, since it contains scaling for adjusting the DC gain [4]. Even though the lossless reconstruction is not attained in implementation, the regularity can be satisfied. However, it is not always satisfied under short word length of coefficients. This report introduces a 'non-lifting form' in implementation of the DWT to structurally guarantee the regularity.

The regularity itself has been analyzed by numerous researchers to improve coding performance of a transform [5-7]. When the regularity is not satisfied, the DWT has the checker board artifact which is observed in a reconstructed signal as unnecessary oscillating noise [5]. It also brings about the DC leakage which decreases the coding gain [6].

The regularity has been structurally guaranteed for the quadrature mirror filter bank [5], the bi-orthogonal filter bank [6] and the DCT [7] respectively. However, since these previous methods are based on the lattice form, these are not directly applicable to the lifting form of the 9-7 DWT. We have investigated the DC lossless condition as a necessary condition to the regularity [8,9]. However it was limited to a constant valued (DC) input signal.

In this report, we implement the lifting DWT in a 'non-lifting' form, instead of the conventional lifting form. Introducing the 'regularity guaranteeing section' composed of (1+z) and (1-z) factors, the DWT always satisfy the regularity in spite of shortening the word length of coefficient values.

Due to this implementation form, the DWT has no oscillation, no checker board artifact or no DC leakage. In the simulation, we confirm that the oscillation is completely suppressed in the new implementation form.

II. IMPLEMENTATION IN 'LIFTING' FORM

Fig.1 illustrates a lifting DWT implemented in the conventional lifting form. Defining a z-transform of a digital signal $x(m)$, m=0,1,$\cdots$, M-1, as

$$X(z)=\sum_{m=0}^{M-1} x(n)z^{-m}, \tag{1}$$

a down sampler and an up sampler affect on a signal as

$$\downarrow 2[X(z)]=\sum_{p=0}^{1} e^{jp\pi}X(z^{1/2}), \quad \uparrow 2[X(z)]=X(z^2) \tag{2}$$

respectively. The filers $H_n(z)$, $n\in\{1,2,3,4\}$ are defined as

$$\begin{bmatrix} H_1(z) & H_3(z) \\ H_2(z) & H_4(z) \end{bmatrix}=\begin{bmatrix} (z+1) & 0 \\ 0 & (1+z^{-1}) \end{bmatrix}\begin{bmatrix} h(1) & h(3) \\ h(2) & h(4) \end{bmatrix} \tag{3}$$

where $h(n)$ denotes a filter coefficient.

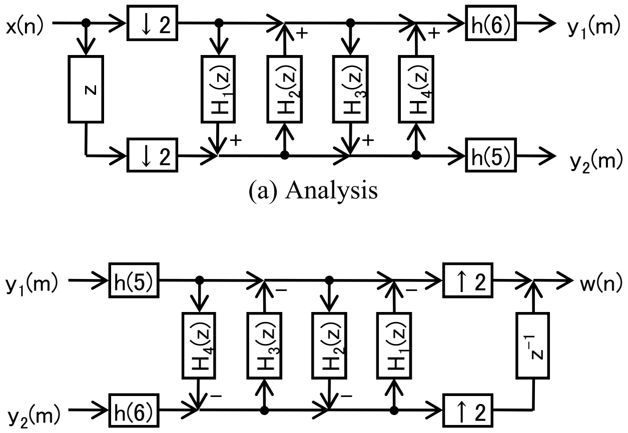
In case of the 5-3 DWT, the coefficient values are given as

$$\begin{bmatrix} h(1) & h(3) & h(5) \\ h(2) & h(4) & h(6) \end{bmatrix}=\begin{bmatrix} -1/2 & 0 & 1 \\ 1/4 & 0 & 1 \end{bmatrix}. \tag{4}$$

As far as the scaling factors $h(5)$ and $h(6)$ are one, this lifting form guarantees the lossless reconstruction. Above $h(1)$ and $h(2)$ also satisfy the regularity. On the contrary, coefficients of the 9-7 DWT are given as

$$\begin{bmatrix} h(1) & h(3) & h(5) \\ h(2) & h(4) & h(6) \end{bmatrix}=\begin{bmatrix} -1.5861 & 0.8829 & 1.2302 \\ -0.0530 & 0.4435 & 0.8129 \end{bmatrix}. \tag{5}$$

These are initially designed as real numbers. However these are truncated to rational numbers with short word length to reduce computational cost in implementation. Then both of the lossless reconstruction and the regularity are not guaranteed in this lifting form.



(a) Analysis

(b) Synthesis

Fig.1 Implementation of DWT in the 'lifting' form.

III.. IMPLEMENTATION IN 'NON-IFTING' FORM

The regularity is explained in the filer bank form. A new implementation form includes a regularity guaranteeing section.

A. Regularity in the Filter Bank Form

Fig.2 illustrates a filter bank equivalent to the DWT in Fig.1. The filters in the figure are related to $H_n(z)$ as

$$\begin{bmatrix} F_1(z) \\ F_2(z) \end{bmatrix} = \begin{bmatrix} h(6) & 0 \\ 0 & h(5) \end{bmatrix} \begin{bmatrix} 1 & H_4(z^2)z^{-1} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ H_3(z^2)z & 1 \end{bmatrix} \begin{bmatrix} 1 & H_2(z^2)z^{-1} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ H_1(z^2)z & 1 \end{bmatrix}. \tag{6}$$

In this form, the regularity is defined as

$$F_1(z)\big|_{z=-1} = 0, \quad F_2(z)\big|_{z=1} = 0 \tag{7}$$

or equivalently

$$F_1(e^{j\omega})\big|_{\omega=\pi} = 0, \quad F_2(e^{j\omega})\big|_{\omega=0} = 0\,. \tag{8}$$

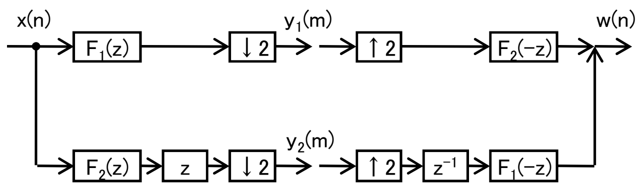


Fig.2 DWT in the 'filter bank' form.

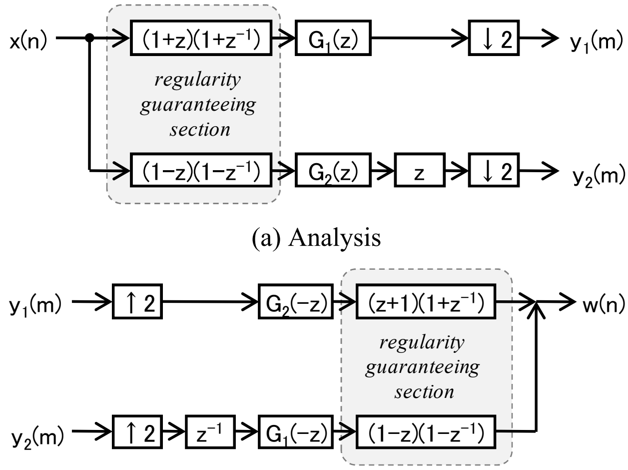
B. Regularity Guaranteeing Section

Since the DWT is initially designed so as to have the regularity (vanishing moment) in eq.(7), we factorize the filters in Eq.(6) as

$$\begin{bmatrix} F_1(z) \\ F_2(z) \end{bmatrix} = \begin{bmatrix} (1+z)(1+z^{-1}) & 0 \\ 0 & (1-z)(1-z^{-1}) \end{bmatrix} \begin{bmatrix} G_1(z) \\ G_2(z) \end{bmatrix}. \tag{9}$$

Fig.3 illustrates the new implementation form based on eq.(9). The regularity guaranteeing section (RGS), which is composed of (1+*z*) and (1-*z*), is separated from other sections.

Even though coefficients of the filters $G_1(z)$ and $G_2(z)$ are truncated, it structurally guarantees the regularity. This is because the factors (1+*z*) and $(1+z^{-1})$ becomes zero at $\omega=\pi$, so as the factors (1-*z*) and $(1-z^{-1})$ at $\omega=0$. It suppresses the checker board affect and the oscillation.



(a) Analysis

(b) Synthesis

Fig.3 DWT in the proposed 'non-lifting' form (type A). It guarantees the regularity.

C. Remaining Section and Other Possible Form

Other remaining sections are derived as follows. Dividing $F_n(z)$ by $G_n(z)$ for n=1,2 respectively, and substituting the property $[F_1(1)\ F_2(-1)] = [1\ 2]$, we have

$$G_1(z) = [1 \quad z+z^{-1} \quad z^2+z^{-2} \quad z^3+z^{-3}] \begin{bmatrix} 1 & -2 & 3 & -4 \\ 0 & 1 & -2 & 3 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} g_1(2) \\ g_1(3) \\ g_1(4) \\ g_1(5) \end{bmatrix} \tag{10}$$

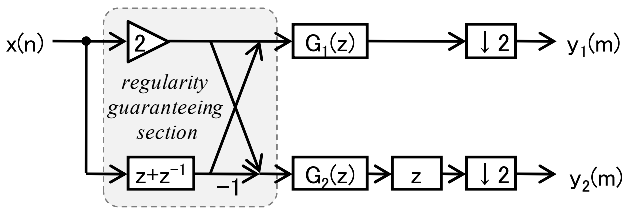
$$G_2(z) = -[1 \quad z+z^{-1} \quad z^2+z^{-2}] \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} g_2(2) \\ g_2(3) \\ g_2(4) \end{bmatrix} \tag{11}$$

where

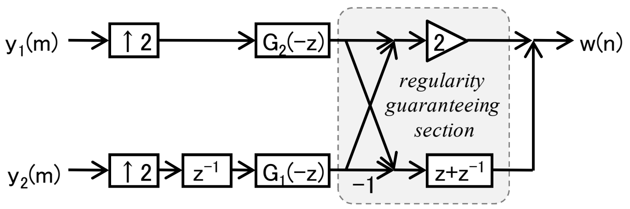
$$\begin{bmatrix} g_1(2) \\ g_1(3) \\ g_1(4) \\ g_1(5) \end{bmatrix} = \begin{bmatrix} h(2) + (1 + 3h(2)h(3))h(4) \\ h(1)h(2) + (h(1) + (1 + 4h(1)h(2))h(3))h(4) \\ h(2)h(3)h(4) \\ h(1)h(2)h(3)h(4) \end{bmatrix},$$

$$\begin{bmatrix} g_2(2) \\ g_2(3) \\ g_2(4) \end{bmatrix} = \begin{bmatrix} h(1) + (1 + 3h(1)h(2))h(3) \\ h(2)h(3) \\ h(1)h(2)h(3) \end{bmatrix}. \quad (12)$$

The coefficients in eq.(12) are shortened for lower computational cost in implementation. Fig.4 illustrates another form of Fig.3. It also guarantees the regularity with simple computational cost.



(a) Analysis



(b) Synthesis

Fig.4 DWT in the proposed 'non-lifting' form (type B).

IV. EXPERIMENTAL RESULTS

Fig.5 illustrates frequency amplitude characteristics of the conventional lifting form in Fig.1 (existing) and the non-lifting form in Fig.4 (proposed), respectively. These are exactly the same before truncation. Table I summarizes coefficient values after truncation. When the word length of the coefficients is 16 [bit] in fixed point expression, truncation errors are hardly observed. However, for 7 or 6 [bit] case, there is a noticeable difference between the two methods. It is quite important to confirm in the figure that the regularity is guaranteed by the non-lifting form.

Fig.6 illustrates a step signal reconstructed from only the low frequency band signal $y_1(m)$ without using the high frequency $y_2(m)$ as an example. When the word length is long enough, there is no difference between them. However, for 6 bit case, it is obviously observed that the conventional lifting form has oscillation in flat region of the step signal. On the contrary, it is clearly indicated that the non-lifting form completely suppresses the oscillation.

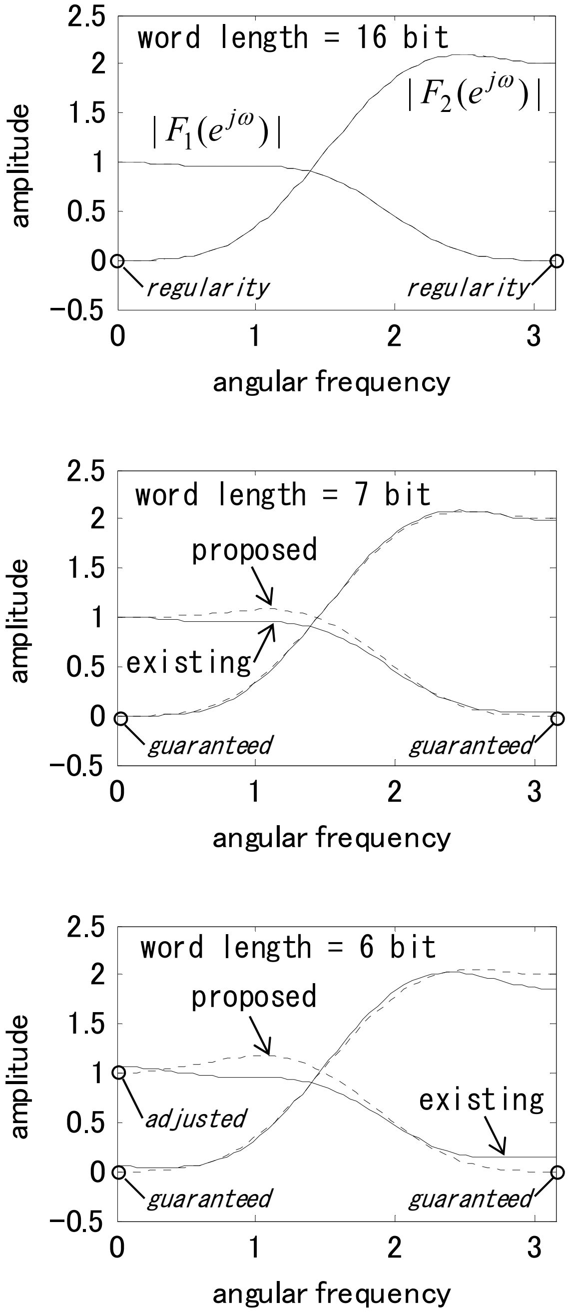


Fig.5 Frequency amplitude characteristics of the DWT.

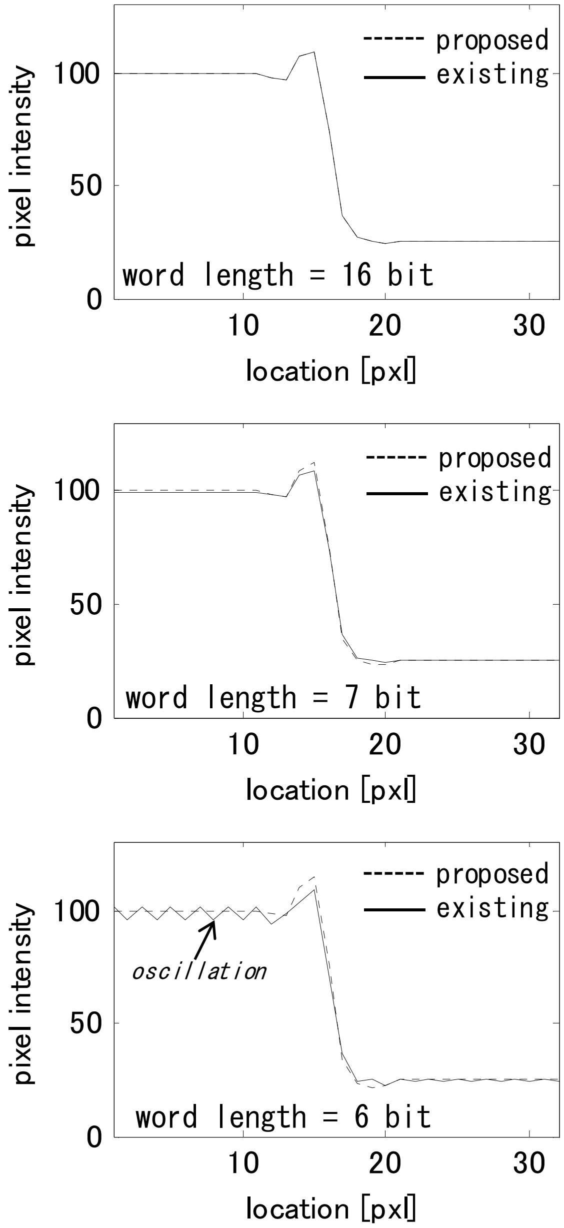


Fig.6 Reconstructed step signals.
Oscillating noise is not observed in the proposed method.

VI. CONCLUSIONS

Effect of a 'non-lifting' form of the lifting DWT on an artifact in a reconstructed signal was investigated. The 'regularity guaranteeing section' was separately implemented. Comparing to the conventional lifting form, it was indicated that the non-lifting form guarantees the regularity in spite of shortening the filer coefficients. It was confirmed that an oscillation was completely suppressed in the new implementation form under short word length.

Table I Shortened values of $h(n)$ in the existing method and $g_1(n)$, $g_2(n)$ in the proposed method.

16 bit						
$h(n)$	-1.5861	-0.0530	0.8829	0.4435	1.2302	0.8129
$g_1(n)$	0.2657	0.0357	-0.0704	0.0267	0	0
$g_2(n)$	-0.4326	0.1250	0.0913	0	0	0
7 bit						
$h(n)$	-1.5938	-0.0547	0.8828	0.4375	1.2266	0.8125
$g_1(n)$	0.2969	0.0313	-0.0781	0.0234	0	0
$g_2(n)$	-0.4375	0.1172	0.0859	0	0	0
6 bit						
$h(n)$	-1.5938	-0.0625	0.8750	0.4375	1.2188	0.8125
$g_1(n)$	0.3125	0.0313	-0.0781	0.0156	0	0
$g_2(n)$	-0.4375	0.1094	0.0781	0	0	0

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