L2 Norm Optimization of Tone Mapping for Two Layer Lossless Coding of HDR Images

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***Abstract*—This paper optimizes a tone mapping function in a backward compatible lossless coding of high dynamic range (HDR) images. The mapping function is designed so that it maximizes quality of decoded low dynamic range (LDR) images. An optimization procedure for a lossy coding system is extended to a lossless system under the condition that a user can specify arbitrary tone mapping function to produce an LDR image from the original HDR image. Superiority of the proposed method on the rate distortion curves of the LDR image was confirmed to be up to 7.5 dB in PSNR improvement for an image sample.**

I. Introduction

Recently, high dynamic range (HDR) images have been attracting researchers' attention since it can express extremely wider dynamic range of pixel values than standard low dynamic range (LDR) images [1]. As it requires huge memory space to be stored, data compression of HDR images is one of important issues. So far, various lossless coding algorithms e.g. RLE, PIZ and LogLuv have been reported [2]–[4]. Those were extended to lossy coding for further compression [5]. In the international standardization activities [6], it is also required to have backward compatibility with the standard coding algorithm such as JPEG and JPEG 2000 developed for LDR images [7].

A backward compatible two layer coding system outputs two kinds of bit streams [8]. One of them contains compressed data for decoding an LDR image. Combining with another bit stream, the input HDR image is decoded. The two layer coding system has been developed also for video signals [9], [10]. Recently, quality of the HDR image was improved introducing piecewise linear modeling of the tone mapping [11]. Replacing the ratio image with a low pass filtered image, coding efficiency was significantly improved [12].

However, those previously reported systems are based on lossy coding of HDR images.

Unlike those previous reports, we deal with lossless coding of HDR images. It is well known that extremely long bit depth makes the coding problem difficult [13]. Introducing a lossless version of the lossy logarithmic range reduction [5], lossless two layer systems were proposed [14]–[16]. Even though a user can use arbitrary tone mapping function to produce an LDR from the HDR image, there is no room to optimize it according to the input image. In this paper, we introduce one more function and determine its shape based on the L2 norm optimization in [17], so that quality of the LDR image is increased.

II. Existing Method

A. High Dynamic Range Images

In this paper, we deal with HDR images in the 'Open EXR' format [2] as an example. In this floating point data format, a pixel value x_H of an input HDR image is expressed as

$$x_H = \begin{cases} x_M \times 2^{-10} \times 2^{x_E - 14} \times (-1)^{xS}, & x_E = 0 \\ (1 + x_M \times 2^{-10}) \times 2^{x_E - 15} \times (-1)^{xS}, & x_E \neq 0 \end{cases} \tag{1}$$

where x_M, x_E and x_S are mantissa, exponent and sign of the pixel. Those are given as integers in the range of $x_M \in [0, 2^{10})$, $x_E \in [0, 2^5)$, and $x_S = 0, 1$. It also defines infinite and not a number. However those are omitted below without loss of generality.

B. Lossy Coding System

Fig. 1 illustrates a backward compatible two layer coding system. Its encoder generates two kinds of bit streams b_{s1} and b_{s2} as compressed data of the input HDR image x_H. The system has backward compatibility as the output LDR image y_L can be reconstructed with a standard decoder from the bit streams b_{s1}. The system also reconstructs the output HDR image y_H combining with another bit streams b_{s2}. Note that both of yL and y_H contain quantization errors since the system is composed of 'lossy' encoders and decoders.

In a previous paper [18], the function h, which maps an HDR pixel value x_H to an LDR pixel value x_L, was optimized according to the histogram of pixel values in the input HDR image. It was designed so that variance of the residue e_H was minimized. It contributes to reduce data size of the bit stream b_{s2}. However, image quality of the LDR image was not considered. Subsequently, a constraint of the LDR image quality was added in [17] to the optimization procedure. As a result, coding performance was significantly improved in lossy coding of the LDR image.

Fig. 2 illustrates a model of Fig. 1 for optimization of h_{ex}. The quantization error e_Q is added by the lossy encoder and

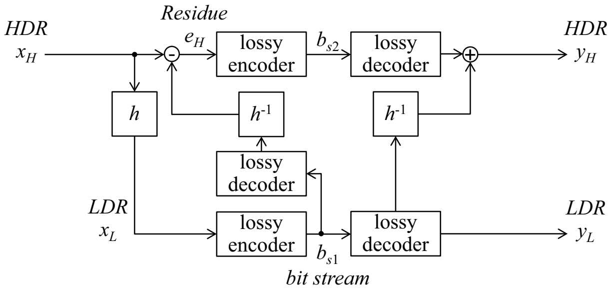

Fig. 1: A two layer backward compatible 'lossy' coding system. The encoder outputs two kinds of bit streams b_{s1} and b_{s2}. The decoder reconstructs an LDR image y_L from b_{s1}, and an HDR image y_H from b_{s1} and b_{s2}. Both of y_L and y_H contain quantization errors.

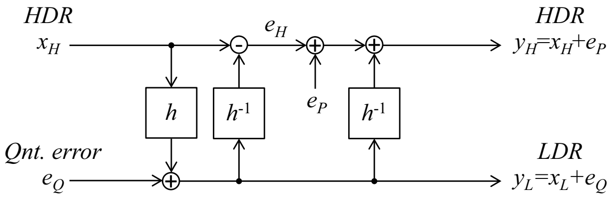

Fig. 2: The mapping h was optimized so that $||e_H||^2$ was minimized to reduce data size of b_{s2} in [18]. A constraint of $||e_Q||^2$ was added to consider quality of y_L in [17].

decoder. In [18], the function h_{ex} was optimized as

$$\hat{h}_{ex} = \arg\min_{h_{ex}} ||e_H||^2 \tag{2}$$

for

$$e_H = h_{ex}^{-1}\left(e_Q + h_{ex}(x_H)\right) - x_H \tag{3}$$

so that data size of b_{s2} in Fig. 1 was reduced. On the contrary, in [17], a constraint of $||e_Q||^2$ with a given constant C_Q was added as

$$\hat{h}_{ex} = \arg\min_{h_{ex}} ||e_H||^2 \quad \text{s.t.} \quad ||e_Q||^2 = C_Q \tag{4}$$

so that data size of b_{s1} in Fig. 1 was reduced maintaining quality of the reconstructed LDR image.

C. Lossless Coding System

Fig. 3 illustrates the lossless coding system reported in [14]. A reversible logarithmic mapping $x_N = f_{ex}(x_H)$ was introduced to reduce the range of HDR pixel values. Its inverse f_{ex}^{-1} reconstructs the rational number x_H from the integer x_N without any loss. Applying a lossless encoder to the residue integer e_H, the HDR pixel value x_H, which is exactly the same as the input value, is reproduced from b_{s1} and b_{s2}.

Similarly to the system in Fig. 1, the LDR pixel value y_L is reconstructed from b_{s1} with a lossy decoder. Note that a mapping g_{ex} was additionally introduced to compensate the effect of f_{ex}. It was designed as

$$g_{ex}(x_Q) = T\left(f_{ex}^{-1}\left(C^{-1}x_Q\right)\right) \quad \text{for} \quad x_L = T(x_H) \tag{5}$$

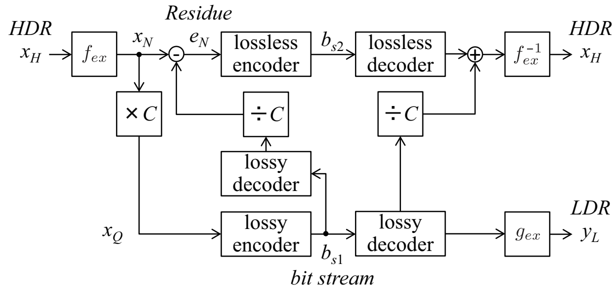

Fig. 3: The existing method in [14]. It reconstructs HDR image x_H without any loss. The reversible logarithmic mapping f_{ex} was introduced to reduce the range of HDR pixel values. The mapping g_{ex} is additionally introduced to compensate the effect of f_{ex}.

for a given tone mapping function T. A constant C was determined to reduce the range of x_N so that x_Q meets requirement of the maximum bit depth of the lossy encoder. Since all of the functions f_{ex}, g_{ex} and a constant C are fixed, this lossless system cannot be optimum unlike the lossy coding system in Fig. .1. Therefore, we introduce an optimization procedure to this existing method in Sec. .III.

III. Propsed Method

A. Proposed Lossless Coding System

In this paper, we develop a backward compatible lossless coding of HDR images expressed in a floating point data format via introducing the optimization procedure in [17] to the existing method in Sec. II-C. The proposed method is illustrated in Fig. 4. The scaling constant C in Fig. 3 is replaced with a function h. We optimize h for a given input HDR image, shown in Sec. III-C.

The mapping from integers to a rational number in (1) is denoted as $x_H = F_{DH}(x_D)$, where $x_D = [x_M, x_E, x_S]$. The proposed function f is defined as

$$x_N = f(x_H) = F_{DN}(F_{DH}^{-1}(x_H)), \tag{6}$$

where

$$F_{DN}(x_D) = (x_M + 2^{10}(x_E - C_{MinE})) \times (-1)^{x_S}.$$

A constant C_{MinE} is the minimum of x_E in a given HDR image. Since the mapping f in (6) is reversible between x_H and x_N, both of range reduction and lossless coding of HDR images become possible [13]–[16]. Note that f approximates a logarithmic function as

$$f(x_H) = 2^{10}(\log_2 x_H + 15 - C_{MinE} - \delta) \tag{7}$$

for

$$\delta = \log_2 \frac{1+\epsilon}{2^\epsilon} \in [0, 0.086], \quad \epsilon = x_M \times 2^{-10} \in [0, 1).$$

We also introduce the function g defined as

$$g(x_Q) = T\left(f^{-1}\left(h^{-1}(x_Q)\right)\right) \quad \text{for} \quad x_L = T(x_H) \tag{8}$$

so that an LDR image x_L, which is tone mapped with a given tone mapping function T, is reconstructed.

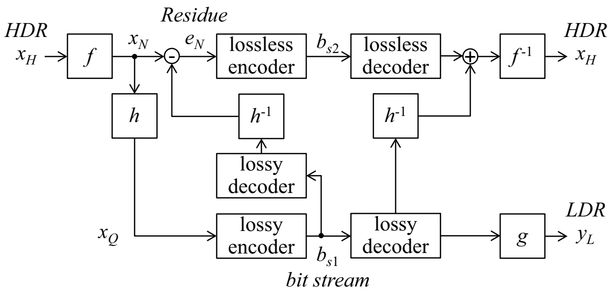

Fig. 4: The proposed method. The scaling constant C in the existing method [14] is replaced with a function h. It is optimized according to histogram of pixel values in the input image.

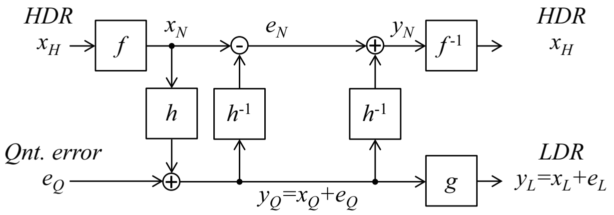

Fig. 5: This paper optimizes h considering f and g so that $||e_L||^2$ is minimized to increase quality of the LDR image y_L.

B. Formulation of the Proposed Optimization Problem

Similarly to [17], we optimize h to increase quality of the reconstructed LDR image. Fig.5 illustrates a model of Fig.4 to determine h for a given tone mapping T in (5). The optimization procedure is formulated as

$$\hat{h} = \arg\min_{h} ||e_L||^2, \tag{9}$$

where

$$e_L = g\left(e_Q + h(f(x_H))\right) - x_L$$

A closed form solution to this problem is described as below. Substituting (8) into (9), we have

$$e_L = T\left(f^{-1}\left(h^{-1}(e_Q + h(f(x_H)))\right)\right) - T(x_H). \tag{10}$$

Applying the Taylor expansion, it can be approximated as

$$e_L = T\left(f^{-1}\left(h^{-1}(e_Q)\right)\right) \simeq T'(f^{-1})'(h^{-1})'e_Q = \frac{T'}{f' \times h'}e_Q, \tag{11}$$

where

$$T' = \frac{dT(x_H)}{dx_H},\ f' = \frac{df(x_H)}{dx_H}, \text{ and } h' = \frac{dh(x_N)}{dx_N}.$$

Therefore, the norm in (9) becomes

$$||e_L||^2 = \int_{\infty}^{\infty}\left(\frac{T'}{f' \times h'}e_Q\right)^2 P_N dx_N \tag{12}$$

where P_N denotes the probability density function (PDF) of x_N. We know that this approximation is ineffective for large noise in dark pixels. However, note that f in (7) reliefs this difficulty comparing to the case without f.

C. Solution to the Optimization Problem

Substituting (12) into the Euler-Lagrange equation

$$\frac{d}{dx_N}\left(\frac{\partial||e_L||^2}{\partial h'}\right) - \frac{\partial||e_L||^2}{\partial h}, \tag{13}$$

we have

$$h' \propto (T')^{2/3}(f')^{-2/3}P_N^{1/3} \tag{14}$$

and therefore

$$h(x_N) = C\int_{-\infty}^{x_N}(T')^{2/3}(f')^{-2/3}P_N^{1/3}dx \tag{15}$$

A scaling parameter C is determined so that (15) satisfies the boundary condition for the minimum and the maximum of pixel values in the image signal. Finally, the optimized function h was derived for a given tone mapping T. Performance of the proposed method is evaluated in Sec. IV.

Note that P_N in (12) can be replaced with P_H which denotes PDF of the HDR image x_H as

$$P_N dx_N = P_H dx_H \text{ or } P_N = P_H\frac{dx_H}{dx_N} = \frac{P_H}{f'}. \tag{16}$$

However we use P_N rather than P_H to avoid unstable experimental results due to the histogram sparseness [19], [20].

IV. Results and Discussions

Fig. 6 illustrates examples of decoded LDR images y_L in our experiments. We used the Hill function

$$x_L = T(x_H) = \left(1 + \left(\frac{x_H}{b \times C_{Geo}}\right)^{-a}\right)^{-1} \tag{17}$$

as the tone mapping function T in (3) where C_{Geo} denotes the geometric mean of x_H [21].

The left column of Fig. 7 shows the LDR image quality index $||e_L||^2$ measured with the peak-signal to noise ratio (PSNR) in the vertical axis, versus lossy coding error index $||e_Q||^2$ in the horizontal axis. Comparing to the case without h, it was observed that the proposed optimization in (15) gives better LDR image quality by approximately 3.5 dB for Mt. Tam West, 3.8 dB for Desk, and 2.0∼2.7 dB for Still Life, respectively. The right column of Fig. 7 indicates the LDR image quality index $||e_L||^2$ in the vertical axis, versus bit rate of the image x_Q in lossy coding (i.e., data size index of the bit stream b_{s1}) in the horizontal axis. Superiority of the proposed method in respect of LDR image quality was observed to be 1.7, 2.6 and 7.5 dB at the maximum, and 0.9, 1.8 and 5.3 dB at the minimum for Mt. Tam West, Desk and Still Life, respectively.

In this paper, variance of the quantization error in the LDR image was considered in the optimization explained in Sec. III-C. It is also possible to minimize the error in the HDR images replacing $||e_L||^2$ in (13) with $||e_N||^2$. Note that reduction of bit rates (data size of the bit streams) were not explicitly considered in this paper. A bit rate constrained approach with iteration can be utilized [22]. Discussions on other norm such as the L infinity can be found in [23].

(a) Mt. Tam West (b) Still Life (c) Dest

Fig. 6: Examples of decoded LDR images.

V. Conclusion

A tone mapping function h was introduced to a two layer lossless coding of HDR images. The function was designed so that it maximizes quality of decoded LDR images according to histogram of pixel values of the input image signal. The function can be used as an option for customization of the system. Applying the proposed method to HDR images in a floating point data format, effectiveness of our optimization was confirmed in PSNR improvement. Since our discussion is limited to monochrome images, it should be extended to color images in the future.

Acknowledgment

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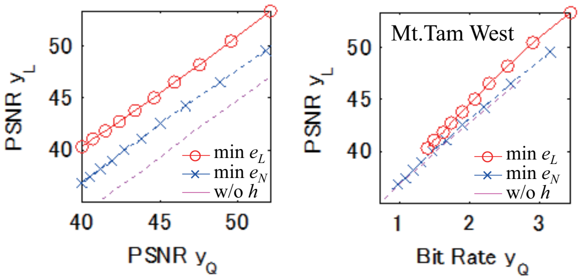
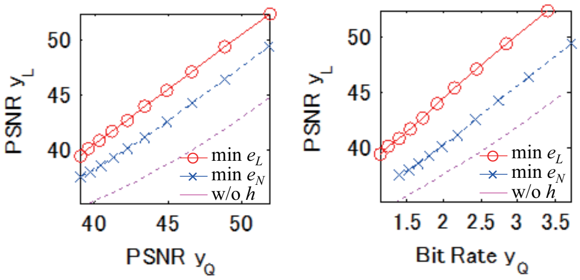
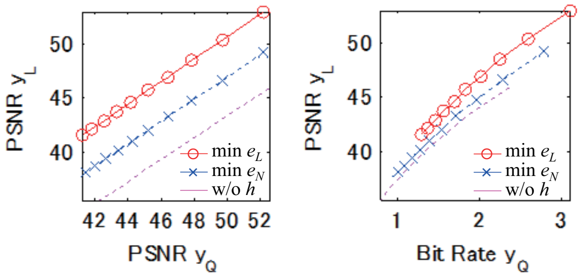


(a) Mt. Tim West



(b) Still Life



(c) Desk

Fig. 7: LDR image quality and Rate distortion curve in the base layer.

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