

# Range Reduction of HDR Images for Backward Compatibility with LDR Image Processing

Masahiro Iwahashi\*, Taichi Yoshida\* and Hitoshi Kiya†

\*Department of Electrical, Electronics and Information Engineering,  
Nagaoka University of Technology, Nagaoka, Niigata, Japan,

†Department of Information and Communication Systems, Faculty of System Design,  
Tokyo Metropolitan University, Hino, Tokyo, Japan

**Abstract**— This paper proposes a new range reduction method with the minimum amount of quantization error in the  $L_2$  norm under the  $L$  infinity norm constraint. It is necessary to reduce dynamic range of pixel values of high dynamic range (HDR) images to have backward compatibility with low dynamic range image processing systems. The simplest approach is to truncate lower bit planes in binary representation of pixel values. However it does not have fine granularity of the reduced range, and also it does not utilize the histogram sparseness. Furthermore, it generates significant amount of quantization errors. In this paper, we propose a new range reduction method which can 1) utilize the histogram sparseness, and also 2) minimize variance of the error 3) under a specified maximum absolute value of the error.

## I. INTRODUCTION

In advanced image and video signal processing technologies, the number of pixels has been increasing to realize extremely high quality video, e.g. the super high vision TV, the 8K broadcasting, etc. The number of frames per second has been also increasing to represent smooth motion of an object. One more factor is the dynamic range of pixel values to represent quite a big number of gradations.

High dynamic range (HDR) images have numerous variations of pixel values comparing to the normal 8 bit depth ( $= 2^8 = 256$ ) gradations [1,2]. Therefore, it is necessary to reduce its dynamic range to feed into a conventional image processing system normally developed for standard low dynamic range images [3-5].

The simplest approach is to truncate a few of lower bit planes in binary representation of pixel values in an HDR image. Truncation from  $H$  bit depth to  $L$  ( $< H$ ) bit depth reduces the dynamic range from  $2^H$  to  $2^L$  where both of  $H$  and  $L$  are integers. More generally, the range reduction for the real numbers  $H$  and  $L$  can be also implemented with the uniform quantization. The latter has finer granularity of reduced range than the former. However, of course, it generates errors. An HDR image reconstructed from the range-reduced LDR image contains errors

In this paper, we discuss how to reduce the errors generated by the range reduction. Amount of the error is evaluated with the  $L_2$  norm and the  $L$  infinity norm. The former is the variance of the errors essentially equivalent to the peak signal to noise ratio (PSNR). The latter is the maximum of the

absolute value of the errors, well utilized in the near lossless coding [6-10]. We consider both of them to design a new range reduction method as inspired by the recent paper [11].

Furthermore, we also consider the histogram sparseness. The histogram (with one pixel value per one histogram bin) tends to be 'sparse' especially for HDR images. It means that not all the pixel value bins are used in an image. Therefore the dynamic range can be reduced with the histogram packing for lossless coding [12-14]. It was extended to lossy coding under the  $L$  infinity constraint [15,16].

Unlike those existing methods, we introduce the  $L_2$  norm sense to minimize the errors. We utilize flexibility of selecting the quantization step size under the  $L$  infinity constraint, to minimize the  $L_2$  norm of the errors. In this paper, we show that the new range reduction method can 1) utilize the histogram sparseness, and 2) minimize variance of the error 3) under a specified maximum absolute value of the error.

## II. EXISTING METHODS

### A. Range Reduction

Fig.1 illustrates the range reduction we are going to discuss in this paper. A pixel value  $x$  of an HDR image is fed into the 'Range Reduction' to output a new pixel value  $y$  of an LDR image. It is processed with a standard image processor designed for LDR images. We define the error as

$$e = x^* - x \quad (1)$$

where  $x^*$  indicates a pixel value reconstructed from  $y$  by the inverse procedure 'Range Expansion'.

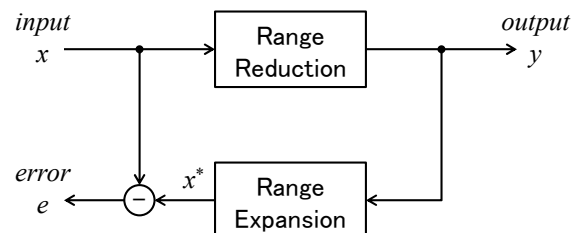


Fig.1 Dynamic range reduction of HDR images.

Our goal is to find how to reduce the dynamic range of  $y$  with small amount of the error  $e$ . We evaluate the error with two criteria. One is the  $L_2$  norm defined as

$$\|e\|_2 = E_{\forall x \in \text{image}} [x^* - x] \quad (2)$$

where  $E[\ ]$  denotes ensemble average of all the pixel values in the image. Based on this criterion, the peak signal to noise ratio (PSNR) is defined as

$$PSNR = 10 \log_{10} \frac{\max\{x\}}{\sqrt{\|e\|_2}} \quad (3)$$

where  $\max\{x\}$  denotes the maximum of pixel values of the image. The other is the  $L$  infinity norm defined as

$$\|e\|_\infty = \max_{\forall x \in \text{image}} \{|x^* - x|\} \quad (4)$$

which means the maximum of the absolute value of the error in the image. In this paper, we consider both of them to design a new method to reduce the dynamic range defined as

$$D(x) = \max\{x\} - \min\{x\} + 1 \quad (5)$$

for all the pixel values  $x$  in the image. Base on this range, the bit depth is defined as

$$B(x) = \log_2(\max\{x\} - \min\{x\} + 1). \quad (6)$$

For example,  $B(x)=8$  [bit] implies  $D(x)=256$ .

### B. Histogram Sparseness

Fig.2(a) illustrates the histogram of an image example ‘Couple’ in SIDBA. The horizontal axis denotes histogram bin (one pixel value per one bin). In case of this image, 52.9 [%] of the histogram bins are ‘zero’. It means that not all the pixel value bins are used in the image. Table I summarizes this ‘histogram sparseness’ for some image examples. Note that the reversible range reduction in [17-20] is applied to the OpenEXR images before evaluating the sparseness.

Especially for HDR images, its histogram tends to be ‘sparse’ as they have longer bit depth than normal LDR images. Therefore this histogram sparseness can be utilized to reduce the dynamic range with the ‘histogram packing’ as illustrated in Fig.3 [12-14].

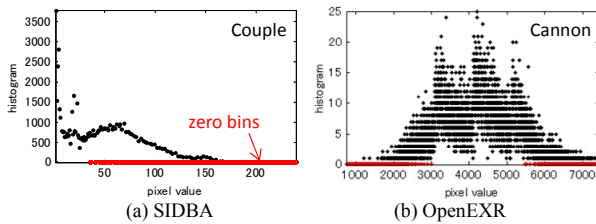


Fig.2 Histogram of the image ‘Couple’.

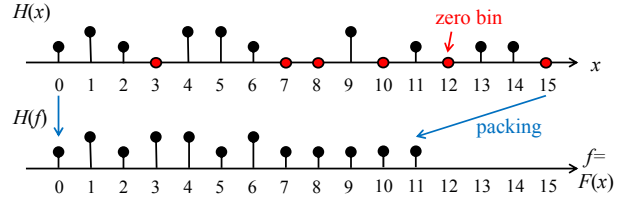


Fig.3 Histogram packing for reversible range reduction.

Table I Histogram sparseness of some image examples.

SIDBA	range	sparseness	OpenEXR	range	sparseness
Girl	7.99	0.528	Tree	14.18	0.678
Couple	7.91	0.529	Still Life	14.51	0.634
Bridge	8.00	0.750	Desk	14.18	0.469
Lena	7.85	0.052	Mt.Tam West	13.72	0.447
Camerman	7.95	0.000	Cannon	12.71	0.355

Table II Comparison of some approaches.

range reduction	range granularity	histogram sparseness	error measure	
			$L_\infty$	$L_2$
(a) Bit-plane Truncation	N.G.	N.A.	Good	fair
<b>(b) Uniform Quantization</b>	Good	N.A.	Good	fair
(c) Lloyd-Max Quantization	Good	N.A.	N.G.	Best
(d) Packing & Quantization	Good	available	N.G.	N.G.
<b>(e) Proposed Method</b>	Good	available	Best	Good

### C. Comparison of Approaches

Table II summarizes some approaches for dynamic range reduction. Method (a) truncates lower bit planes of pixel values of the original  $H$  planes remaining upper  $L$  planes for integers  $H$  and  $L$ . As a result, its dynamic range is reduced from  $2^H$  to  $2^L$ . More generally, method (b) is applicable for real numbers  $H$  and  $L$ . The latter has finer granularity of reduced range than the former. Both of them can specify the  $L$  infinity norm of the errors in (4) beforehand by setting an arbitrary quantization step size. However those are not optimum in  $L_2$  norm sense.

Method (c) is the best in  $L_2$  norm sense. However it can't specify the  $L$  infinity norm beforehand. In addition, all of the methods (a), (b), (c) do not utilize the histogram sparseness to reduce the dynamic range.

Method (d) denotes applying the histogram packing first, and then the uniform quantization. It can utilize the histogram sparseness. However, its inverse procedure magnifies the error value, and therefore it can't specify the  $L$  infinity norm beforehand ( $L$  infinity norm constraint).

In this paper, we introduce method (e) which can satisfy the  $L$  infinity norm constraint, and also minimize the  $L_2$  norm of the errors under the constraint in dynamic range reduction of images.

#### D. Existing Method

Fig.4 illustrates the method (b) in table II (existing method). For example, when the quantization step size is set to  $\varepsilon=3$ , the bins  $\{0,1,2\}$  are grouped into a quantization block  $X_0$ . Similarly,  $X_1=\{3,4,5\}$ ,  $X_2=\{6,7,8\}$ , etc. All of them have the same number of elements. The quantized value  $y$  is set to

$$y = \begin{cases} \lfloor x/\varepsilon \rfloor, & x \geq 0, \\ -\lfloor -x/\varepsilon \rfloor, & x < 0. \end{cases} \quad (7)$$

where  $\lfloor \cdot \rfloor$  denotes flooring into integer. Its inverse procedure is given as

$$x^* = \begin{cases} y \cdot \varepsilon + \delta, & y \geq 0 \\ y \cdot \varepsilon - \delta, & y < 0 \end{cases} \quad (8)$$

for

$$\delta = \begin{cases} (\varepsilon-1)/2, & \varepsilon \in \text{odd}, \\ \varepsilon/2, & \varepsilon \in \text{even}. \end{cases} \quad (9)$$

Obviously, its  $L$  infinity norm in (4) can be specified to be less than  $\delta$  beforehand. However, it does not utilize the histogram sparseness, and also the  $L2$  norm in (2) is not considered.

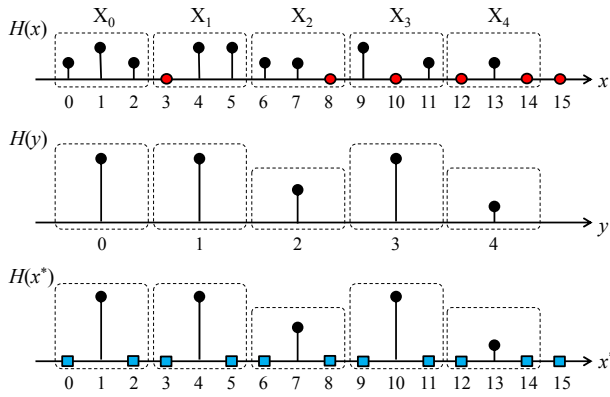


Fig.4 Uniform quantization.

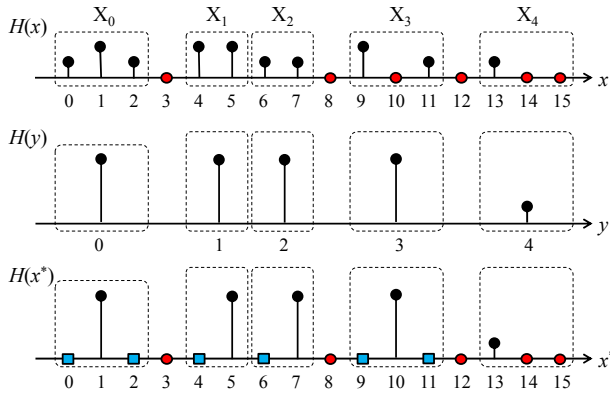


Fig.5 The proposed method.

#### III. PROPOSED METHOD

Fig.5 illustrates the method (e) in table II (proposed method). Unlike the existing method, zero bins are skipped in the process of starting the quantization block. Namely, denoting a quantization block as

$$X_y = \{x \mid (x \geq s_y) \wedge (x \leq e_y)\} \quad (10)$$

with the starting point  $s_y$  and the ending point  $e_y$  for  $y=0,1,\dots$ , the starting point is set to

$$s_y = \min\{x \mid (x > e_{y-1}) \wedge (H(x) \neq 0)\}. \quad (11)$$

Since the term ' $H(x) \neq 0$ ' skips the zero bins, the histogram sparseness is effectively utilized for range reduction.

Secondly, we set the ending point to be

$$e_y < s_y + \varepsilon \quad (12)$$

so that the  $L$  infinity norm of the error does not exceed a specified value  $\delta$  in (9). Thirdly, we also consider the  $L2$  norm minimization under the  $L$  infinity norm constraint. It is realized by adding one more inequality

$$e_y < s_y + \varepsilon', \quad \varepsilon' = \left\{ x \mid \sum_{u=s_y}^x \{P(u)\}^{1/3} < Th \right\} \quad (13)$$

for

$$\begin{cases} P(x) = H(x) \cdot \left\{ \sum_{x=\min\{x\}}^{\max\{x\}} H(x) \right\}^{-1} \\ Th = \frac{\varepsilon}{\max\{x\} - \min\{x\} + 1} \sum_{x=\min\{x\}}^{\max\{x\}} \{P(u)\}^{1/3} \end{cases} \quad (14)$$

where  $Th$  is set according to [21]. As a result of setting the ending point  $e_y$  under (12) and (13), the proposed method can 1) utilize the histogram sparseness, and 2) minimize variance of the error 3) under a specified maximum absolute value.

#### IV. EXPERIMENTAL RESULTS

Fig.6 compares the methods in table II. The horizontal axis of Fig.6(a) indicates the bit depth in (6) of the input image 'Couple'. The horizontal axis indicates the bit depth of the error  $e$ . It is essentially equivalent to the  $L$  infinity norm in (4). Under this norm, method (c) labeled as 'LloydMax' and method (d) labeled as 'Pack&Qnt' are not good as summarized in table II. This tendency is also true for 'Cannon' and 'Mt.TamWest' in Fig.6(c) and Fig.6(e). It was observed that the proposed method has smaller error in the  $L$  infinity norm in case of reducing the range by less than 2 [bit].

The horizontal axis of Fig.6(b) indicates PSNR in (3). It is essentially equivalent to the  $L2$  norm in (2). Under this norm, method (c) labeled as 'LloydMax' is the best as summarized in table II. This tendency is not true for 'Cannon' and 'Mt.TamWest' in Fig.6(d) and Fig.6(f). It is conceivable that

the summation in (13) does not reach the threshold  $Th$  as far as  $P(u)=0$  at zero bins, resulting in generating huge  $\varepsilon'$  for extremely sparse cases. For these images, the proposed method was observed to be the best.

According to Fig.6(b) for 'Couple', method (c) is the best and method (d) is the second. However these methods are not good because those do not satisfy the  $L$  infinity norm constraint. Therefore, excluding those methods, the proposed method was found to be the best.

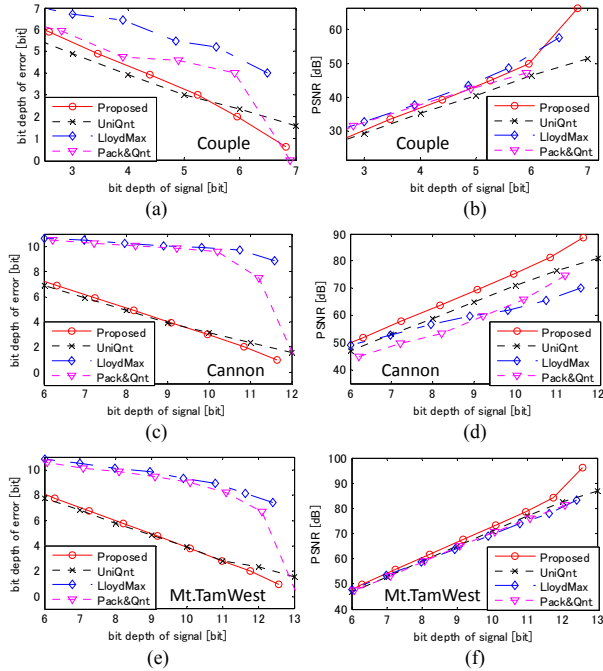


Fig.6 Evaluation results.

## V. CONCLUSIONS

In this paper, we proposed a new range reduction method. It can specify the maximum absolute error value beforehand. Introducing the zero-bin skipping procedure in (11), our method can utilize the histogram sparseness. Our method can also increase the PSNR introducing the  $L2$  norm minimization in (13). Since our discussion is limited to single component images, it should be extended to color images in the future.

This work was supported by JSPS KAKENHI Grant Number 26289117.

## REFERENCES

- [1] E. Reinhard, G. Ward, S. Pattanaik, P. Debevec, W. Heidrich, K. Myszkowski, "High Dynamic Range Imaging - Acquisition, Display and Image based Lighting," Morgan Kaufmann, 2010.
- [2] Y. Zhang, D. Agrafiotis, D. R. Bull, "High Dynamic Range image & video compression a review," International Conference on Digital Signal Processing, pp.1-7, July 2013.

- [3] R. Xu, S. N. Pattanaik, C. E. Hughes, "High-Dynamic-Range Still Image Encoding in JPEG 2000," IEEE Computer Graphics and Applications, vol.25, no.6, pp.57-64, 2005.
- [4] G. Ward, and M. Simmons, "JPEG-HDR: A Backwards-Compatible, High Dynamic Range Extension to JPEG," Thirteenth Color Imaging Conference, Nov. 2005.
- [5] M. Iwahashi, H. Kiya, "Error Equalization for High Quality LDR Images in Backward Compatible HDR Image Coding", Asia-Pacific Signal and Information Processing Association 2011 Annual Summit and Conference (APSIPA), OS.10, IVM.5, no.2, pp.1-4, Oct. 2013.
- [6] ISO/IEC 14495-1: 1999(E), "Information Technology - Lossless and Near-Lossless Compression of Continuous-tone still Images: Baseline," Dec. 1999.
- [7] X. Wu and P. Bao, "L (infinity) Constrained High-Fidelity Image Compression via Adaptive Context Modeling," IEEE Trans. Image Processing, vol.9, no.4, pp. 536-542, 2000.
- [8] A.Bazhyna, K.Egiazarian, "Lossless and Near Lossless Compression of Real Color Filter Array Data," IEEE Trans. Consumer Elec., vol.54, issue 4, pp.1492-1500, Nov. 2008.
- [9] S.E.Qian, M.Bergeron, I.Cunningham, L.Gagnon, A.Hollinger, "Near Lossless Data Compression Onboard a Hyperspectral Satellite," IEEE Trans. Aerospace and Electronic Systems, Vol.42, Issue 3, 851-866, 2006.
- [10] J.Taquet, C.Labit, "Hierarchical Oriented Predictions for Resolution Scalable Lossless and Near-Lossless Compression of CT and MRI Biomedical Images," IEEE Trans. Image Processing, Vol.21, Issue 5, pp.2641-2652, 2012.
- [11] S. Chuah, S. Dumitrescu, X. Wu, "L2 Optimized Predictive Image Coding with L infinity Bound," IEEE Trans. Image Processing, Vol.22, Issue 12, pp.5271-5281, Dec. 2013.
- [12] A. J. Pinho, "An Online Preprocessing Technique for Improving the Lossless Compression of Images with Sparse Histograms," IEEE Signal Processing Letters, vol.9, no.1, pp.5-7, 2002.
- [13] P. J. S. G. Ferreira, and A. J. Pinho, "Why Does Histogram Packing Improve Lossless Compression Rates?," IEEE Signal Processing Letter, vol. 9, no. 8, pp. 259-261, Aug. 2002.
- [14] M. Aguzzi and M. Albanesi, "A Novel Approach to Sparse Histogram Image Lossless Compression using JPEG2000," Electronic Letters on Computer Vision and Image, 2006.
- [15] M. Iwahashi, H. Kobayashi, and H. Kiya, "Lossy Compression of Sparse Histogram Image," IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP), no.IVMS-P10.5, pp.1361-1364, March, 2012.
- [16] M. Iwahashi, H. Kobayashi, H. Kiya, "Fine Rate Control and High SNR Coding for Sparse Histogram Images," Picture Coding Symposium (PCS), pp.205-209, May 2012.
- [17] James F. Blinn, "Floating -Point Tricks," IEEE Computer Graphics and Applications, pp.81-84, August 1997.
- [18] M. Iwahashi, H. Kiya, "Two Layer Lossless Coding of HDR Images", IEEE International Conference on Acoustics, Speech, and Signal Processing (ICASSP), pp.1340-1344, May 2013.
- [19] Chew Yin Ping, T. Shibata, M. Iwahashi, H. Kiya, "Lossless Bit Depth Scalable Coding for Floating Point Images", International Workshop on Advanced Image Technology (IWAIT), 2C-5, pp.169-174, Jan. 2013.
- [20] M. Iwahashi, H. Kiya, "Efficient Lossless Bit Depth Scalable Coding for HDR Images," APSIPA Annual Summit and Conference, OS.49-IVM.17, Dec. 2012.
- [21] S. Lloyd, "Least Squares Quantization in PCM," IEEE Trans. on Information Theory, vol.28, no.2, pp. 129- 137, Mar. 1982.