Optimum SPT Allocation for Multipliers of Minimum Lifting 2D Wavelet Transform

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***Abstract*— Implementation issues of the separable 2D lifting wavelet transform has been discussed under the international standard JPEG 2000 scheme. Recently, the minimum lifting step structure of the transform was proposed introducing the multi-dimensional memory accessing. It can reduce latency in a parallel processing platform. However, its output signal is distorted due to truncation of multiplier coefficient values. Especially, a few coefficients are extremely sensitive to the truncation in this structure. Therefore a special treatment is required to these coefficients. In this paper, a tolerable (the maximum) truncation error is optimally assigned to each coefficient under a given cost function introducing the noise gain. It is applied to expressing coefficient values in the sum-of-power-of-two (SPT) format for reducing computational cost of multipliers of the transform.**

Keywords— JPEG 2000, compression, image coding, coefficient, rounding noise

I. Introduction

Especially after the JPEG 2000 international standard adopted the lifting wavelet transform [1], various implementation issues have been discussed on this transform [2-4]. The JPEG 2000 defines two types of filters 5/3 and 9/7 for the lossless coding and the lossy coding of images, respectively [4]. Both of them are composed of cascaded lifting steps. Those are defined as the one-dimensional (1D) signal processing and applied to an image signal vertically and horizontally. As a result, the total processing is expressed as a separable (Sep) two-dimensional (2D) processing.

Recently, non-separable (Nsp) 2D structures were reported for increasing coding performance of the transform [5-8]. Utilizing its freedom in diagonal direction, adaptive prediction schemes were proposed in [5,6]. However they have no compatibility with the JPEG 2000. The Nsp 2D structure in [7,8] minimized the total number of lifting steps. The compatibility is maintained in [7]. Since a lifting step must wait for a result of its previous lifting step, the more the lifting steps exist, the more the delay increases. Therefore the minimum lifting step structure can reduce latency of the transform in a parallel processing platform.

However, the output signal of the minimum lifting step structure is distorted due to truncation of multiplier coefficient values. Especially, a few coefficients are extremely sensitive to the truncation in this structure. On the contrary, some other coefficients are relatively tolerant. Therefore a special treatment to each coefficient is expected to decrease implementation cost of the minimum lifting structure.

As an implementation cost [9], the word length of coefficients is used in this paper. It is closely related to the sum-of-power-of-two (SPT) format [10-14]. This format is reported to be beneficial for designing multiplier-less adaptive filters [11,12]. Different number of SPT terms are allocated to each coefficient value under a given total number of SPT terms to reduce implementation cost in [13].

Inspired by [13], this paper proposes an optimization procedure to determine the tolerable (the maximum) truncation error (tolerance) for each coefficient of the minimum lifting structure. In this optimization process, the noise gain (sensitivity) of each coefficient is taken into account for an input signal with colored spectrum. Unlike the tabu-search method in [14], our method is simple and stable with satisfactory performance for this case.

In our experiments, effectiveness of the proposed method in lossy coding is investigated. The tolerance is assigned to each coefficient so that coding performance of the transform is maintained. It is indicated that the proposed method can reduce the word length cost and the total number of SPT terms maintaining the same lossy coding performance.

II. Non-Separable 2D Wavelet Transform

Nsp 2D minimum lifting structure and its multiplier coefficients to be expressed in SPT format are summarized.

A. Minimum Lifting Structure

Figure 1 illustrates the Sep 2D structure of the forward 9/7 wavelet transform. Pixel values $x(n_1,n_2)$ at location n_1, n_2 of the input image is divided into 4 groups as

$$\begin{cases} x_1(\mathbf{m}) = x(2m_1,2m_2), & x_2(\mathbf{m}) = x(2m_1,2m_2+1), \\ x_3(\mathbf{m}) = x(2m_1+1,2m_2), & x_4(\mathbf{m}) = x(2m_1+1,2m_2+1), \end{cases} \tag{1}$$

for $\mathbf{m}=(m_1, m_2)$. In the first step, x_3 is predicted from x_1 as

$$X_3^{(1)}(\mathbf{z}) = X_3(\mathbf{z}) + h_1 V_1(\mathbf{z}) X_1(\mathbf{z}) \tag{2}$$

where

$$X_b(\mathbf{z}) = \sum_{\forall m_1,m_2} x_b(m_1,m_2) z_1^{-m_1} z_2^{-m_2} \tag{3}$$

for $c \in \{1,2,3,4\}$, $\mathbf{z}=(z_1,z_2)$ and $V_1(\mathbf{z})=(1+z_1)$. Simultaneously, x_4 is predicted from x_2 in the same manner. In (2), h_1 denotes a multiplier coefficient. Having the result of the first step, updating in the second step can start. As illustrated in Figure 1, it has 8 lifting steps in total where

$$\begin{cases} V_1(\mathbf{z}) = V_3(\mathbf{z}) = 1+z_1, & V_2(\mathbf{z}) = V_4(\mathbf{z}) = 1+z_1^{-1}, \\ H_1(\mathbf{z}) = H_3(\mathbf{z}) = 1+z_2, & H_2(\mathbf{z}) = H_4(\mathbf{z}) = 1+z_2^{-1}, \end{cases} \tag{4}$$

and coefficient values are defined as listed in Table I [1].

Figure 2 illustrates the Nsp 2D structure of the forward transform [7,8]. Unlike the structure in Figure 1, x_4 is predicted from x_1, x_2 and x_3 in the first step. In the second step, x_2 and x_3 are updated from x_1 and x_4 simultaneously. In total, this minimum lifting structure has 6 steps which is reduced from 8 to 6 (75 %). Figure 3 illustrates the backward transform in this structure. The forwardly transformed signals Y_1, Y_2, Y_3 and Y_4 are backwardly transformed to reconstruct the original input signals (from right to the left in Figure 3). This paper focuses on expressing coefficients of this backward transform in SPT format for reducing computational complexity of the decoder.

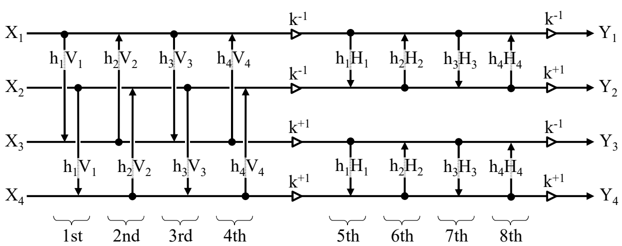


Figure 1 The separable (Sep) 2D structure of the 9/7 forward wavelet transform in JPEG 2000 standard. This structure has 8 lifting steps.

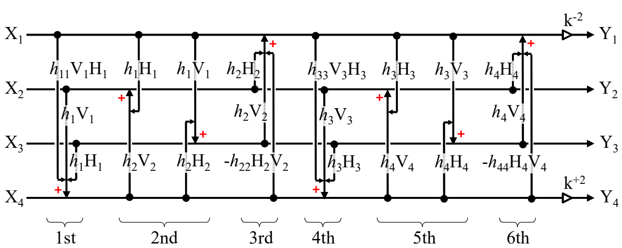


Figure 2 The non-separable (Nsp) 2D minimum-lifting structure of the forward transform. The number of lifting steps is reduced from 8 to 6.

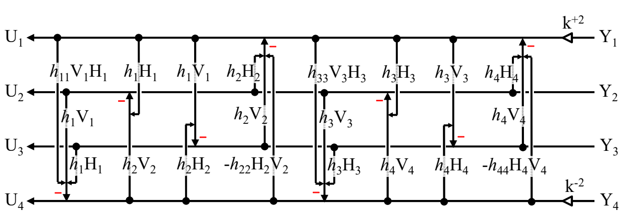


Figure 3 The backward transform of the non-separable (Nsp) structure. In this paper, coefficients are expressed in the Sum-of-Power-of-Two format.

Table I Coefficient values of the 2D wavelet transform with 9/7 filters.

c	coefficient		c	coefficient	
1	h_1	−1.586134342059924	6	h_{22}	0.0028068929640050
2	h_2	−0.052980118572961	7	h_{33}	0.7795319672951906
3	h_3	0.882911075530934	8	h_{44}	0.1966983278099528
4	h_4	0.443506852043971	9	k^{+2}	1.5133283284009633
5	h_{11}	2.515822151061868	10	k^{-2}	0.6607951369394080

B. Truncation of Coefficients

For reducing computational complexity, coefficient values are truncated and expressed with short word length binary or the SPT format. For example, when an original coefficient value h_c is truncated as

$$h'_c = \lfloor h_c \cdot 2^{+W_c} \rfloor \cdot 2^{-W_c}, \tag{5}$$

the truncated coefficient h'_c has W_c [bit] in its fraction part in binary expression. In this case, the truncation error takes values in the range of

$$\Delta h_c = h_c - h'_c \in [0, 2^{-W_c}). \tag{6}$$

The shorter the word length W_c is, the larger the truncation error Δh_c becomes. This paper uses the word length as the implementation cost. Given the word length or the tolerable truncation error, h_c is expressed in the SPT format. For example, h_1= $-(1.100101100000\cdots)_2$ is truncated to h'_1 = $-(1.1001)_2$ at W_c=4 and 5, respectively (the same result). In SPT format, it is expressed with 3 terms as

$$h'_1 = -(2^0 + 2^{-1} + 2^{-4}). \tag{7}$$

Note that less number of terms is preferable for low computational cost [13].

III. Proposed Method

The proposed method allocates different word length (or number of SPT terms) to each coefficient so that the word length cost is reduced under the same coding performance.

A. Effect of the Truncation on Signals

Firstly, we investigate effect of truncating each coefficient value on the reconstructed signal (output signal of the backward transform) in Figure 3. The effect is measured as

$$I_c = \left(\frac{1}{\#\mathbf{M}} \sum_{\forall \mathbf{m}} \sum_{b=1}^{4} \{ u_b(\mathbf{m}|c) - x_b(\mathbf{m}) \}^2 \right)^{1/2} \tag{8}$$

for $c \in \{1,2,\cdots,N\}$ where $\#\mathbf{M}$ denotes the total number of pixels. It indicates the standard deviation (SD) of the noise due to the truncation. In (8), $u_b(\mathbf{m}|c)$ denotes pixel values of the output signals U_b in Figure 3 in which only the coefficient number c is truncated. When all the coefficients are truncated,

$$I_{\forall c} = \left(\frac{1}{\#\mathbf{M}} \sum_{\forall \mathbf{m}} \sum_{b=1}^{4} \{ u_b(\mathbf{m}|\forall c) - x_b(\mathbf{m}) \}^2 \right)^{1/2} \tag{9}$$

is used instead of (8). Relation between (8) and (9) is described as

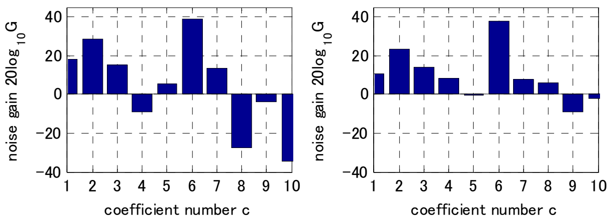
$$I_{\forall c}^2 = \sum_{c=1}^{N} I_c^2 + \beta \tag{10}$$

where N is the total number of coefficients. When the noise $u_b(\mathbf{m}|c)$ are uncorrelated each other, β becomes zero.

Secondly, we define the noise gain $G(c)$ of the coefficient number c as

$$G(c) = I_c \cdot \sigma_x^{-1} \cdot \Delta h_c^{-1} \tag{11}$$

where σ_x denotes SD of pixel values of the input image. Figure 4 illustrates the measured noise gain for a 2D AR(1) model signal with correlation ρ. It is observed that the coefficient number 6 has the largest value of G, whereas the number 10 has the least effect for ρ=0.99 case.



(a) ρ=0.99 (b) ρ=0.00

Figure 4 The noise gain G(c) of the coefficient number c in log scale. The coefficient number 6 has the largest effect on the output signal.

B. Optimum Word Length Assignment

The proposed method optimally assigns the tolerance Δh_c in (11) for each coefficient. We define the word length cost as

$$J = \frac{1}{N} \sum_{c=1}^{N} W_c \ \text{[bit]} \tag{12}$$

where W_c denotes the word length of a coefficient number c. The optimization problem of this paper is described as

$$\min J \quad s.t. \quad I_{\forall c}^2 = \varepsilon \quad \wedge \quad \max\{W_c\} = W \tag{13}$$

for a given word length W. The parameter ε is set to the variance in (10) of the existing method, so that the proposed method does not change the lossy coding performance of the existing method. From (10), (11), (12) and $-\log_2(\Delta h_c) > W_c$ from (6), the problem in (13) is expressed with the tolerance Δh_c as

$$\begin{aligned} \Delta\hat{h}_c = \underset{\Delta h_c}{\arg\max}\ & \log_2\left(\prod_{c=1}^{N} \Delta h_c\right)^{1/N} \\ & + \lambda_1\left(\sum_{c=1}^{N} G^2(c)\sigma_x^2 \Delta h_c^2 + \beta - \varepsilon\right) + \lambda_2\left(\min\{\Delta h_c\} - 2^{-W}\right) \end{aligned} \tag{14}$$

where λ_1 and λ_2 are Lagrange multipliers. As a solution to this problem, we assign the tolerance for each coefficient as

$$\Delta h_c = \left(\frac{\max\{G(c)\}}{G(c)}\right)^{\alpha} \cdot 2^{-W}, \quad \alpha \in [0,1] \tag{15}$$

where α is a hyper parameter to be determined experimentally. Note that α=0 means the existing method.

IV. Experimental Results

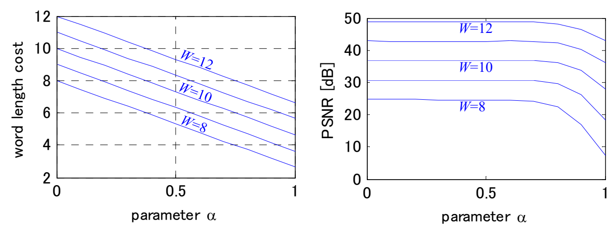
It is experimentally confirmed that the proposed method reduces the word length cost under the same coding performance of the existing method.

A. Selection of the Hyper-Parameter

Our solution in (15) is simple and stable comparing to a numerical optimization procedure. Instead, it includes the hyper parameter α. Figure 5(a) indicates that the word length cost J in (12) decreases as α becomes close to 1. In contrast, as indicated in Figure 5(b), the peak signal to noise ratio:

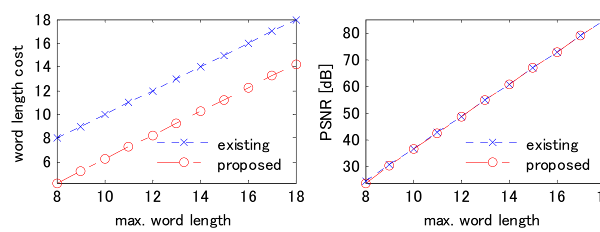
$$PSNR = 10(\log_{10} 255^2 - \log_{10} I_{\forall c}^2) \quad [dB] \tag{16}$$

decreases for α greater than approximately 0.7. Therefore we suggest to set to α=0.7. Figure 6(a) indicates that the proposed method decreases the word length cost by approximately 4 [bit] for a given word length W. Figure 6(b) indicates that the proposed method has the same PSNR as the existing method. Figure 7 illustrates the rate-distortion curves in lossy coding at W=12. It was observed that the proposed method keeps coding performance of the existing method under the lower word length cost. Note that the PSNR ceiling (approximately 49 [dB] in Figure 7(b)) is increased by 6 [dB] per 1 [bit] increase of W. It's up to the capacity of the implementation platform.



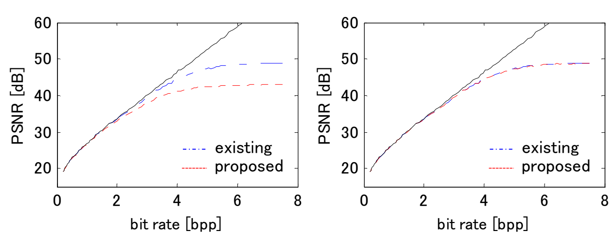
(a) (b)

Figure 5 The parameter α>0 is determined so that the proposed method has the least length cost under the same PSNR as the existing method with α=0.



(a) (b)

Figure 6 The proposed method with α=0.7 has less word length cost under the same PSNR comparing to the existing method with α=0.



(a) α=1.0, ρ=0.9 (b) α=0.7, ρ=0.9

Figure 7 The proposed method with α=0.7 has the same performance in the rate-distortion curve as the existing method.

B. Expression of Coefficients in SPT format

In the following experiments, the parameters are set to W=12 and α=0.7 as an example. Table II summarizes the assigned word length W_c for each coefficient. Each of them satisfies W_c< $-\log_2(\Delta h_c)$ where the tolerance Δh_c is derived by the assignment in (15). For example, the tolerance of the existing method is Δh_c= 2^{-12} for all coefficients. Since h'_1= -(1.1001 0110 0000)$_2$ in 12 [bit] and h'_1= -(1.1001 011)$_2$ in 7 [bit] are the same, W_c is counted as 7 in the Table. It was observed that the word length cost is reduced to 71.8 [%] by the proposed method. Under these tolerances, the coefficient values are expressed in SPT format. Results are summarized in Table III. Table IV summarizes the total number of the SPT terms. It is reduced to 74.5 [%] by the proposed method. Note that the number of terms can be decreased by setting smaller value of the word length W at the cost of lower ceiling of PSNR (illustrated in Figure 7).

Table II The word length W_c of each coefficient (example for W=12).

	$-h_1$	$-h_2$	h_3	h_4	h_{11}	h_{22}
Existing	7	12	7	9	6	12
Proposed	7	9	7	4	6	12
	h_{33}	h_{44}	k^{+2}	k^{-2}	total	
Existing	9	12	11	11	96	100 %
Proposed	9	4	7	3	68	71.8%

Table III Coefficients in the SPT format (example for W=12).

	Existing method	Proposed Method
$-h_1$	$2^0 +2^{-1} +2^{-4} +2^{-6} +2^{-7}$	$2^0 +2^{-1} +2^{-4} +2^{-6} +2^{-7}$
$-h_2$	$2^{-5} +2^{-6} +2^{-8} +2^{-9} +2^{-12}$	$2^{-5} +2^{-6} +2^{-8} +2^{-9}$
h_3	$2^{-1} +2^{-2} +2^{-3} +2^{-7}$	$2^{-1} +2^{-2} +2^{-3} +2^{-7}$
h_4	$2^{-2} +2^{-3} +2^{-4} +2^{-8} +2^{-9}$	$2^{-2} +2^{-3} +2^{-4}$
h_{11}	$2^{+1} +2^{-1} +2^{-6}$	$2^1 +2^{-1} +2^{-6}$
h_{22}	$2^{-9} +2^{-11}+2^{-12}$	$2^{-9} +2^{-11}+2^{-12}$
h_{33}	$2^{-1} +2^{-2} +2^{-6} +2^{-7} +2^{-8} +2^{-9}$	$2^{-1} +2^{-2} +2^{-6} +2^{-7} +2^{-8} +2^{-9}$
h_{44}	$2^{-3} +2^{-4} +2^{-7} +2^{-10} +2^{-12}$	$2^{-3} +2^{-4}$
k^{+2}	$2^0 +2^{-1} +2^{-7} +2^{-8} +2^{-10} +2^{-11}$	$2^0 +2^{-1} +2^{-7}$
k^{-2}	$2^{-1} +2^{-3} +2^{-5} +2^{-8} +2^{-11}$	$2^{-1} +2^{-3}$

Table IV The total number of SPT terms (example for W=12).

	$-h_1$	$-h_2$	h_3	h_4	h_{11}	h_{22}
Existing	5	5	4	5	3	3
Proposed	5	4	4	3	3	3
	h_{33}	h_{44}	k^{+2}	k^{-2}	total	
Existing	6	5	6	5	47	100 %
Proposed	6	2	3	2	35	74.5%

V. Conclusions

In this paper, a simple and stable allocation method of the word length of coefficients of the minimum lifting 2D wavelet transform was proposed. It was experimentally confirmed that the proposed method reduces the word length cost keeping the same PSNR as the existing method in lossy coding. An allocation example for 12 bit maximum word length case was indicated in SPT format. Since the analysis in this paper is limited to the truncation of coefficients, the rounding of signal values should be also considered as described in [15].

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